

1983

# Optimal and maximal taxation within and among industries with various degrees of competitiveness

Jim William Paulsen  
*Iowa State University*

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>

 Part of the [Economics Commons](#)

## Recommended Citation

Paulsen, Jim William, "Optimal and maximal taxation within and among industries with various degrees of competitiveness " (1983).  
*Retrospective Theses and Dissertations*. 8428.  
<https://lib.dr.iastate.edu/rtd/8428>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact [digirep@iastate.edu](mailto:digirep@iastate.edu).

## INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.
2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.
3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of "sectioning" the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.
4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.
5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.

**University  
Microfilms  
International**

300 N. Zeeb Road  
Ann Arbor, MI 48106



8323309

**Pausen, Jim William**

OPTIMAL AND MAXIMAL TAXATION WITHIN AND AMONG INDUSTRIES  
WITH VARIOUS DEGREES OF COMPETITIVENESS

*Iowa State University*

PH.D. 1983

**University  
Microfilms  
International** 300 N. Zeeb Road, Ann Arbor, MI 48106



Optimal and maximal taxation within and among industries  
with various degrees of competitiveness

by

Jim William Paulsen

A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of the  
Requirements for the Degree of  
DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University  
Ames, Iowa

1983

## TABLE OF CONTENTS

	page
CHAPTER I. INTRODUCTION	1
CHAPTER II. THE GENERAL OPTIMAL TAXATION PROBLEM	4
A Perfectly Competitive Economy	4
An Imperfectly Competitive Economy	9
CHAPTER III. PERFECT COMPETITION	21
Taxation and Perfect Competition	21
The Optimal Taxation Path	26
Optimal Taxation Schemes	32
The General Equilibrium Problem	34
CHAPTER IV. MONOPOLISTIC COMPETITION	36
Price and Nonprice Competition	37
The Optimal Taxation Path	41
Optimal Taxation Schemes	50
Nonprice competition	51
Price competition	57
Summary: Monopolistic Competition	63
CHAPTER V. MONOPOLY	65
The Optimal Taxation Path	65
Optimal Taxation Schemes	70
CHAPTER VI. SUMMARY OF CONCLUSIONS	75
BIBLIOGRAPHY	78
ACKNOWLEDGMENTS	80

## CHAPTER I. INTRODUCTION

The optimal commodity tax system is the one which minimizes the aggregate loss of taxpayers' well-being for any given amount of tax revenue. The first analytical formulation and solution of the problem appears in the celebrated article by Ramsey [19]. This original analysis considered the problem according to the criterion of economic efficiency. Recently, the theory has been extended to take account of both distributional and efficiency considerations [4, 9, 17]. Most analyses have been limited to consideration of single excise taxes levied in an economy which consists only of perfectly competitive industries. As a result, the major focus has been on the optimal configuration of excise rates among industries. That is, whether a uniform or a differential excise rate structure is required [3, 10, 11, 19, 20, 21, 22].

This paper examines the optimal taxation of an economy when both perfectly and imperfectly competitive industries exist together. In such a world, there is no configuration of single excise rates which satisfies the optimal commodity taxation problem. Rather, a combination of taxes is generally required for optimal taxation of imperfectly competitive industries. Therefore, in this paper, the major focus is on the type of tax or combination of taxes required for optimal taxation of industries with various degrees of competitiveness. The choice of tax instruments is expanded to include license fees (L) in addition to unit (t) and *ad valorem* (s) excise taxes. Excise taxes enter the analysis in the usual manner as differences between prices paid by buyers and the net receipts of sellers. License fees are treated as taxes on the existence of firms in an industry.



Licenses can either be sold at a fixed price to a market-determined number of firms or a fixed number can be auctioned off at a market-determined price. If the market for licenses is perfectly competitive, both methods of selling have the same impact on the industry equilibrium. Consequently, for expository efficiency, we will assume that licenses are issued at a fixed price per firm. License fees are similar to, but different from, lump-sum taxes. Like lump-sum taxes, they are a fixed amount which is independent of the size of the firm, the choice of inputs, and the quantity and price of output, but, unlike a truly lump-sum tax, license fees can be avoided by going out of business. The optimal taxation of three types of industries is analyzed: a perfectly competitive industry, a monopolistically competitive industry, and a single-plant monopolist. Within monopolistic competition, there are two distinct industry equilibria considered. Chamberlin [8] noted the differences between equilibria with and without price competition. The former is characterized by a large number of relatively small firms, each of which perceives that its own price changes have no impact on rivals. That is, each firm believes it can pursue an independent pricing policy. In contrast, nonprice competition is characterized by firms who correctly realize that their rivals will react to their actions.

Chapter II introduces the basic properties of optimal commodity taxation as discussed by Ramsey, and extends his analysis to include the determination of the conditions which must be satisfied if the economy consists of both perfectly and imperfectly competitive industries. It is shown that optimal taxation of the economy requires each particular industry to be moved along its optimal path. Therefore, each type of competitive

structure is considered separately in Chapters III-V. While a long-run partial-equilibrium approach is utilized, the results are easily extended to the general (economy-wide) optimal taxation problem. For each market structure, the characteristics of its optimal path are described. That is, the optimal adjustment of output per-firm, industry sales, and the number of firms, as tax collections are increased, is determined. The tax or combination of taxes which move each industry along this optimal path is then derived. Maximal taxation schemes are also examined. There has been surprisingly little analysis of the maximum revenue potential of single excise taxes (except for papers by Bishop [6] and Adams [1]) and (to our knowledge) no analysis of the revenue potential of multiple taxes applied to specific commodities. Finally, Chapter VI briefly summarizes our conclusions.

## CHAPTER II. THE GENERAL OPTIMAL TAXATION PROBLEM

In a first-best world, pareto optimality requires marginal cost pricing and this is satisfied, in the absence of taxation, by perfectly competitive markets. Imagine a public sector that has a fixed revenue constraint which must be satisfied by taxation. With the exception of lump-sum taxes which do not affect the marginal conditions, taxation generally distorts pareto optimality. Therefore, barring the use of lump-sum taxes, the public sector must choose that particular combination of commodity taxes levied at appropriate rates so that the resulting welfare loss from the distortions away from pareto optimality is minimized for any required collections level. This is the optimal taxation problem, which was first formally analyzed by Ramsey. His basic conclusion can be explained by considering the following model.

## A Perfectly Competitive Economy

Suppose an economy produces  $n$  commodities,  $x_i$  ( $i=1\dots n$ ), in perfectly competitive markets. Let  $P_i(x_i)$  represent the inverse demand function for the  $i^{\text{th}}$  commodity (where  $P_i$  is the price of commodity  $i$ ). Then, define social utility ( $U$ ) as the sum of the gross benefits (measured in terms of the numeraire,  $y$ ) yielded by consumption of all goods, or

$$(II-1) \quad U = \sum_{i=1}^n \int_0^{x_i} P_i(k_i) dk_i + y.$$

Consumers' equilibria require that their total endowment,  $z$  (measured in units of  $y$ ) must be exhausted either on  $y$  or on outlays for all  $x_i$  consumed.

The outlays are the sum of the total resource cost of each commodity ( $c_i(x_i)$ ) and total taxes paid ( $\bar{R}$ ). That is,

$$(II-2) \quad z = y + \sum_{i=1}^n c_i(x_i) + \bar{R} \quad \text{or} \quad y = z - \sum_{i=1}^n c_i(x_i) - \bar{R}.$$

Substituting  $y$  from equation (II-2) into equation (II-1) yields the social welfare function ( $W$ ) which the public sector desires to maximize for any given collections level,  $\bar{R}$ . That is,

$$(II-3) \quad W = \sum_{i=1}^n \int_0^{x_i} P_i(k_i) dk_i - \sum_{i=1}^n c_i(x_i) + z - \bar{R}.$$

Social welfare is defined by equation (II-3) assuming the public sector squanders the tax revenue. That is, the use of  $\bar{R}$  by the public sector does not affect social welfare. Alternatively, one can assume that  $\bar{R}$  is rebated in a lump-sum fashion, in which case

$$(II-3a) \quad W = \sum_{i=1}^n \int_0^{x_i} P_i(k_i) dk_i - \sum_{i=1}^n c_i(x_i) + z.$$

While the choice between these methods of revenue disposal do imply different levels of social welfare, the difference does not affect the marginal conditions and, therefore, the conclusions concerning optimal taxation. Total tax collections are equal to the sum of the  $n$  industries' gross (of tax) profits provided all markets are characterized by free entry so that net (of tax) supply prices are driven to average costs. Therefore,

$$(II-4) \quad \bar{R} = \sum_{i=1}^n \left[ P_i(x_i)x_i - c_i(x_i) \right].$$

The problem, then, is to determine a taxation scheme which satisfies (II-4) and maximizes (II-3a). We can formulate the problem in terms of the following Lagrangean function:

$$(II-5) \quad \mathcal{L} = \sum_{i=1}^n \int_0^{x_i} P_i(k_i) dk_i - \sum_{i=1}^n c_i(x_i) + z \\ + \lambda \left[ \bar{R} - \sum_{i=1}^n (P_i(x_i) x_i - c_i(x_i)) \right].$$

Let  $mc_i$  and  $mr_i$  represent respectively the marginal cost and marginal revenue of commodity  $i$ . Then, (II-5) is maximized when

$$(II-5a) \quad \frac{\partial \mathcal{L}}{\partial x_i} = P_i - mc_i - \lambda (mr_i - mc_i) = 0 \quad \text{for } i=1 \dots n,$$

$$\text{and (II-5b) } \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - \sum_{i=1}^n [P_i(x_i) x_i - c_i(x_i)] = 0.$$

Equations (II-5a) and II-5b) can be solved for the optimal values of each industry's output and  $\lambda$  (i.e.,  $x_1^* \dots x_n^*$  and  $\lambda^*$ ). While these equations provide the optimal output levels and the optimal value of the constraint, there is no mention of tax rates here. However, the particular optimal taxation scheme is that which induces each industry to choose the appropriate output ( $x_i^*$ ) and yields  $\lambda^*$ . Once the values of  $x_i^*$  and  $\lambda^*$  are known, the appropriate taxation scheme can be derived from the industry's (behavioral) equilibrium conditions. Moreover, the basic properties of the optimal taxation scheme are defined by equations (II-5a) and (II-5b).

Equation (II-5b) simply limits the choice to those taxation schemes which can satisfy the revenue constraint ( $\bar{R}$ ). The  $n$  equations of (II-5a) provide two additional characteristics of the optimal taxation scheme. First, for any particular industry  $j$ ,

$$(II-6) \quad P_j - mc_j = \lambda(mr_j - mc_j),$$

or the deviation between price and marginal cost as a result of taxation must be proportional to the deviation between marginal revenue and marginal cost. Essentially, the marginal welfare loss ( $P_j - mc_j$ ) must be proportional to the marginal increase in tax collections ( $mr_j - mc_j$ ). This demonstrates the tradeoff or second best nature of optimal commodity taxation. That is, social welfare must be reduced as collections are increased from zero to their maximum. Equation (II-6) also demonstrates the basic Ramsey conclusion. He related this result to the price elasticity of demand for any given commodity. Note that

$$(II-6a) \quad mr_j = P_j + \frac{\partial P_j}{\partial x_j} x_j \quad \text{or} \quad mr_j = P_j \left(1 + \frac{1}{e_j}\right)$$

where  $e_j$  is the price elasticity of demand for good  $x_j$ . Substituting this expression for  $mr_j$  into equation (II-6) and rearranging yields

$$(II-6b) \quad \frac{P_j - mc_j}{P_j} = \frac{\theta}{e_j} \quad \text{where} \quad \theta = \frac{\lambda}{1-\lambda}.$$

This represents an alternative expression of the Ramsey conclusion. That is, the optimal percentage deviation between the price and marginal cost of any commodity will vary inversely with the commodity's price elasticity of demand. This implies that larger deviations between price and marginal

cost are required for commodities whose demands are relatively inelastic. Equation (II-6) can also be manipulated to show that, within a perfectly competitive economy, optimal taxation requires equal proportionate reductions of all industries' outputs below the no-tax equilibrium. Such reductions require relatively higher tax rates on commodities with relatively inelastic demands.

Essentially, equation (II-6) defines the optimal taxation path for the  $j^{\text{th}}$  industry. As the collections constraint is increased, the taxation scheme must adjust  $x_j$  so that the equation continues to be satisfied. That is, optimal taxation must move the  $j^{\text{th}}$  industry along its optimal path by maintaining the appropriate proportion between the marginal welfare loss and the marginal collections gain. Equation (II-6) must similarly be satisfied by all other industries in the economy. That is, each industry must be moved along its particular optimal path. Since all  $n$  equations of (II-5a) must be satisfied simultaneously,

$$(II-7) \quad \lambda = \frac{P_i - mc_i}{mr_i - mc_i} \quad \text{for all } i=1\dots n.$$

This describes the final characteristic of the optimal taxation scheme; namely, it must equate the proportion ( $\lambda$ ) between the marginal welfare loss and the marginal collections gain across all  $n$  industries. Essentially, equation (II-6) defines the optimal taxation path for each particular industry and equation (II-7) defines the optimal taxation path for the entire economy. Consequently, the entire taxation scheme not only must satisfy the revenue constraint (equation (II-5b)) and move each particular industry to its optimal path (equation (II-6)), but it must also move each industry

to a particular point on its optimal path so that  $\lambda$  is equated across all industries (equation (II-7)).

Much of the literature on optimal commodity taxation has emphasized the demand aspects of the problem. This is exemplified by Ramsey's conclusion concerning the demand elasticity and optimal tax rates. Supply-side responses have received very little attention. Commonly, analysis is limited to excise taxes in perfectly competitive markets, assuming fixed producer prices [3, 11, 17, 19, 20, 21]. Under these assumptions, the major issue has been whether a uniform or a differential (excise) rate structure is required and conclusions have been largely dependent on demand assumptions.

#### An Imperfectly Competitive Economy

This paper emphasizes the supply aspects of the problem by considering taxation in both perfectly and imperfectly competitive markets. It removes the assumption of fixed producer prices and allows for the use of license fees in addition to excise taxes. It considers whether optimal commodity taxation may require different tax instruments (e.g., licenses) or a combination of taxes. The emphasis here is not on whether uniform or differential rates among industries are required. Rather, the focus is within industries and the issue is what tax or combination of taxes are required to move industries, characterized by various types of competitive behavior, to and along their optimal paths. While previous analyses have been mainly concerned with the optimal taxation path of the entire economy, emphasis here is on the optimal taxation path within particular industries. For each market structure analyzed, that taxation scheme which moves the



industry along its optimal taxation path is determined. Once it is known how each industry can be moved along its optimal path, the general (economy-wide) solution across all industries can easily be satisfied. In comparison, then, while earlier analysis has been concerned with the optimal configuration of rates among industries, this paper considers the optimal tax or combination of taxes within industries.

Specifically, this paper will consider the optimal taxation of four distinct industries: a perfectly competitive industry, both price and non-price monopolistically competitive industries, and a monopoly. Let  $P_1(Q_1)$  and  $P_4(Q_4)$  represent the inverse demand function for the perfectly competitive industry's good and the monopolist's good, respectively. Assume the perfectly competitive industry consists of a large number of firms,  $n_1$ , each of which produces  $q_1$  units of output and equilibrium ensures  $Q_1 = n_1 q_1$ . Similarly, let  $P_2(Q_2, n_2)$  and  $P_3(Q_3, n_3)$  represent the inverse demand functions for the "composite" goods of the two monopolistically competitive industries. The number of firms ( $n_i$ ) is an argument in these demand functions because the level of variety (i.e., each firm produces a distinct product under conditions of monopolistic competition) affects consumers' valuations of the quantity of the composite good consumed. Total social utility (U), therefore, is simply the sum of the gross benefits (measured in terms of the numeraire, y) yielded by consumption of all goods, or

$$(II-8) \quad U = \int_0^{Q_1} P_1(k_1) dk_1 + \int_0^{Q_2} P_2(k_2, n_2) dk_2 + \int_0^{Q_3} P_3(k_3, n_3) dk_3 \\ + \int_0^{Q_4} P_4(k_4) dk_4 + y.$$

Consumers' equilibria require that their total endowment,  $z$  (measured in units of  $y$ ) must be exhausted either on  $y$  or on outlays for all  $Q_i$  consumed. These outlays consist of the sum of the total production cost of each good and total taxes paid ( $\bar{R}$ ). Let  $q_i$  ( $i=1\dots3$ ) represent output per firm and  $c_i(q_i)$  ( $i=1\dots3$ ) represent total production cost per firm. Then, total production cost for each industry is simply  $n_i c_i(q_i)$  ( $i=1\dots3$ ), or total production cost per firm times the number of firms in the industry. Alternatively, since  $Q_i = n_i q_i$  ( $i=1\dots3$ ), total industry production cost can be rewritten as  $n_i c_i\left(\frac{Q_i}{n_i}\right)$ . Assuming a single-plant monopolist, his total production cost is represented by  $c_4(Q_4)$ . Combining all this information implies

$$(II-9) \quad z = y + \sum_{i=1}^3 n_i c_i\left(\frac{Q_i}{n_i}\right) + c_4(Q_4) + \bar{R}$$

if collections are squandered, while, if collections are rebated in a lump-sum fashion, then,

$$(II-9a) \quad z = y + \sum_{i=1}^3 n_i c_i\left(\frac{Q_i}{n_i}\right) + c_4(Q_4).$$

Assume collections are rebated, solve (II-9a) for  $y$ , and substitute into (II-8). This yields the social welfare function which is simply the sum of the consumer surpluses across all industries plus  $z$ , or

$$(II-10) \quad B = \int_0^{Q_1} P_1(k_1) dk_1 - n_1 c_1\left(\frac{Q_1}{n_1}\right) + \int_0^{Q_2} P_2(k_2, n_2) dk_2 - n_2 c_2\left(\frac{Q_2}{n_2}\right) \\ + \int_0^{Q_3} P_3(k_3, n_3) dk_3 - n_3 c_3\left(\frac{Q_3}{n_3}\right) + \int_0^{Q_4} P_4(k_4) dk_4 - c_4(Q_4) + z.$$

Let  $B_i$  represent the consumers' surplus or total social welfare attributable to the consumption of the  $i^{\text{th}}$  good; then,

$$(II-10a) \quad B = \sum_{i=1}^4 B_i + z.$$

Tax collections extracted from both the perfectly competitive and monopolistically competitive industries will equal the gross-of-tax profits, since free entry is assumed. Assuming a profits tax of 100% is always levied on the monopolist, total economy-wide collections can be expressed as the sum of the gross (of tax) profits of the industries. Let  $\pi_i$  represent gross industry profit for industry  $i$ . Let  $\bar{R}$  represent the required collection level, so that

$$(II-11) \quad \bar{R} - \sum_{i=1}^4 \pi_i = 0.$$

The economy-wide optimal taxation scheme consists of those taxes required to maximize (II-10a) while satisfying (II-11). That is, the appropriate taxation scheme will maximize the following Lagrangean function:

$$(II-12) \quad \mathcal{L} = B_1(Q_1, n_1) + B_2(Q_2, n_2) + B_3(Q_3, n_3) + B_4(Q_4) + z \\ + \lambda \left\{ \bar{R} - [\pi_1(Q_1, n_1) + \pi_2(Q_2, n_2) + \pi_3(Q_3, n_3) + \pi_4(Q_4, n_4)] \right\}.$$

Before proceeding, it is useful to compare equation (II-12) with equation (II-5). Note that, in equation (II-12),  $\mathcal{L}$  is determined by the output levels of each industry, the number of firms existing in each industry, and  $\lambda$ . In equation (II-5),  $\mathcal{L}$  was determined only by the output level of each industry and  $\lambda$ . The number of firms affects both total resources cost and enters explicitly into the demand functions of monopolistically competitive

industries. Therefore, not only must the level of sales be appropriately adjusted, but the number of firms existing in the industry must also be adjusted for optimal taxation. Moreover, previously it was assumed that all industries were perfectly competitive. Therefore, that type of tax(es) which optimally adjusts any given industry will also optimally adjust the remaining industries. Since all industries are identically competitive, they will all require the same general tax scheme. The only difference will be the rates at which the particular taxes are levied. Alternatively, in equation (II-12), various types of competitive structure exist simultaneously. Because of this, each industry may react differently to the same tax. That is, a separate taxation policy may be required for each of the four different competitive structures. The inclusion of various market structures, therefore, changes the major focus of optimal taxation. Rather than determining a single optimal tax policy for the entire economy and focusing on the appropriate rates to levy in each industry, the major focus is on the optimal tax policy for each type of competitive industry.

Consider the conditions which must be satisfied to maximize  $\mathcal{L}$  in equation (II-12). Let  $P_i$  represent the price in the  $i^{\text{th}}$  industry,  $mr_i(Q_i)$  ( $mr_i(n_i)$ ) represent the marginal revenue of industry sales (of an additional firm) in industry  $i$ , and let  $mc_i(Q_i)$  ( $mc_i(n_i)$ ) represent the marginal cost of industry sales (of an additional firm) in industry  $i$ .

Then, if

$$(II-12a) \quad \frac{\partial \mathcal{L}}{\partial Q_1} = P_1 - mc_1(Q_1) - \lambda [mr_1(Q_1) - mc_1(Q_1)] = 0$$

$$(II-12b) \quad \frac{\partial \mathcal{L}}{\partial n_1} = -mc_1(n_1) + \lambda [mc_1(n_1)] = 0$$

$$(II-12c) \quad \frac{\partial \mathcal{L}}{\partial Q_2} = P_2 - mc_2(Q_2) - \lambda [mr_2(Q_2) - mc_2(Q_2)] = 0$$

$$(II-12d) \quad \frac{\partial \mathcal{L}}{\partial n_2} = \int_0^{Q_2} \frac{\partial P_2(k_2, n_2)}{\partial n_2} dk_2 - mc_2(n_2) - \lambda [mc_2(n_2) - mc_2(n_2)] = 0$$

$$(II-12e) \quad \frac{\partial \mathcal{L}}{\partial Q_3} = P_3 - mc_3(Q_3) - \lambda [mr_3(Q_3) - mc_3(Q_3)] = 0$$

$$(II-12f) \quad \frac{\partial \mathcal{L}}{\partial n_3} = \int_0^{Q_3} \frac{\partial P_3(k_3, n_3)}{\partial n_3} dk_3 - mc_3(n_3) - \lambda [mr_3(n_3) - mc_3(n_3)] = 0$$

$$(II-12g) \quad \frac{\partial \mathcal{L}}{\partial Q_4} = P_4 - mc_4(Q_4) - \lambda [mr_4(Q_4) - mc_4(Q_4)] = 0$$

$$(II-12h) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - [\pi_1(Q_1, n_1) - \pi_2(Q_2, n_2) - \pi_3(Q_3, n_3) - \pi_4(Q_4)] = 0$$

are simultaneously satisfied, equation (II-12) is maximized for  $R = \bar{R}$ . These 8 equations provide the optimal values of each industry's sales level, number of firms, and  $\lambda$  (i.e.,  $Q_i^*$ ,  $n_i^*$ , and  $\lambda^*$  for  $i=1\dots 4$ ). The optimal tax policy for each industry can be derived from the particular industry's (behavioral) equilibrium conditions. Note that equations (II-12a) to (II-12h) can be subdivided into the optimal conditions that must hold for each industry. For example, (II-12a) and (II-12b) represent the conditions which must hold if the perfectly competitive industry is optimally taxed.

Previously,  $\lambda$  was interpreted as the ratio of the marginal welfare loss to the marginal collections gain of an additional unit of industry sales ( $Q_i$ ). Since the number of firms affects both welfare and collections,  $\lambda$  must also equal the ratio of the marginal welfare loss to the marginal collections gain of an additional firm in each industry.<sup>1</sup>

The entire set of equations defines the optimal taxation path for the entire economy. However, they are only simultaneously satisfied if each particular industry's conditions are satisfied. That is, the economy is on its optimal path only if each separate industry is moved along its respective optimal path. Consequently, before the economy can be moved along its optimal path, the public sector must know how to optimally tax each particular industry. Our analysis, therefore, will consider each industry separately. Once optimal tax policies for each are known, optimal taxation for the entire economy is simply a matter of using these separate policies so that (II-12a)-(II-12h) are simultaneously satisfied (i.e., so that  $\lambda$  is equated across all industries).

Specifically, the optimal taxation path for a given industry,  $j$ , will be derived by maximizing the social welfare from consumption of the  $j^{\text{th}}$  good (i.e.,  $B_j$  from equation (II-10a)) subject to a given level of revenues collected from the industry. For example, the optimal taxation path for the perfectly competitive industry is defined by maximizing the following Lagrangean function:

---

<sup>1</sup>From equation (II-12b), it may appear that  $\lambda=1$  always, but this is not true because  $MC_1(n_1)=0$  at the optimum.

$$(II-13) \quad \mathcal{L}_c = \int_0^{Q_1} P_1(k_1) dk_1 - n_1 c_1 \left( \frac{Q_1}{n_1} \right) + \lambda_1 \left[ \bar{R}_1 - \left( P_1(Q_1) Q_1 - n_1 c_1 \left( \frac{Q_1}{n_1} \right) \right) \right], \text{ or}$$

$$(II-13a) \quad \frac{\partial \mathcal{L}_c}{\partial Q_1} = P_1 - mc_1(Q_1) - \lambda \left( mr_1(Q_1) - mc_1(Q_1) \right) = 0$$

$$(II-13b) \quad \frac{\partial \mathcal{L}_c}{\partial n_1} = -mc_1(n_1) + \lambda \left[ mc_1(n_1) \right] = 0$$

$$(II-13c) \quad \frac{\partial \mathcal{L}_c}{\partial \lambda_1} = \bar{R}_1 - \left( P_1(Q_1) Q_1 - n_1 c_1 \left( \frac{Q_1}{n_1} \right) \right) = 0.$$

Note that (II-12a) = (II-13a) and (II-12b) = (II-13b). That is, the tax policy which satisfies (II-13a) and (II-13b) for all values of  $\bar{R}_1$  will yield the tax policy required to satisfy equations (II-12) and (II-12b) for all values of  $\bar{R}$ . A similar analysis will be performed for each market structure. The tax policies which move all industries along their particular optimal paths are those which are then required to maximize equation (II-12) for the entire economy. Consequently, while each industry is considered separately (in a partial-equilibrium framework), the results are easily extended or applicable to the general equilibrium optimal taxation problem where industries of various competitive structures exist together.

Because of the role played by the Lagrangean multiplier ( $\lambda$ ) in characterizing the optimal taxation path, it is worthwhile to consider its intuitive interpretation and its numerical sign. To simplify, assume an economy which produces two goods under conditions of perfect competition.

If  $x$  is the only good which is taxed, then optimal taxation requires maximizing

$$(II-14) \quad \mathcal{L} = \int_0^x P(k)dk - c(x) + z + \lambda \left[ \bar{R} - (P(x)x - c(x)) \right]$$

or

$$(II-14a) \quad \frac{\partial \mathcal{L}}{\partial x} = P - mc - \lambda(mr - mc) = 0$$

and

$$(II-14b) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - (P(x)x - c(x)) = 0.$$

Equation (II-14a) defines the optimal value of  $\lambda$ ,

$$(II-15) \quad \lambda = \frac{P - mc}{mr - mc},$$

but what exactly does  $\lambda$  represent? A common feature of Lagrange functions is that the Lagrange multiplier is simply the change in the objective function due to a marginal change in the constraint. For our purposes, the objective function is social welfare, or

$$(II-16) \quad B = \int_0^x P(k)dk - c(x) + z$$

and the constraint is tax collections, or

$$(II-17) \quad \bar{R} = P(x)x - c(x).$$

Therefore,  $\lambda$  is simply the change in social welfare due to a marginal change in tax collections (i.e.,  $\lambda = \frac{\partial B}{\partial \bar{R}}$ ).

Next, consider the numerical sign of  $\lambda$ . Essentially, we need to know whether  $\bar{R}$  and  $B$  move directly or inversely with one another. Since both



are functions of output ( $x$ ), their relationship can be determined by examining their relationship to output. This is illustrated in Figure II-1. Examination of equation (II-16) shows that  $B$  increases with  $x$  until it is maximized when  $P=mc$  at point  $w$  (i.e., when  $x = x_B$ ). Similarly, from equation (II-17),  $\bar{R}$  is maximized when  $mr = mc$  at point  $T$ . This occurs at an output level ( $x_R$ ) less than that where  $P = mc$  (i.e.,  $x_B > x_R$ ) since  $P > mr$  at any  $x$ . Moreover, note that  $B$  exceeds  $R$  at every output level, since the latter is included in the former.

The sign of  $\lambda$  over the full range of output levels can be determined using equation (II-15). For output levels less than  $x_R$ ,  $P > mr > mc$  so that  $\lambda > 0$ . Similarly, for any output level in excess of  $x_B$ ,  $mc > P > mr$  or  $\lambda > 0$ . Finally,  $\lambda$  is negative whenever output is between  $x_R$  and  $x_B$  since  $P > mc$  while  $mc > mr$ .

Note that the optimal taxation problem is only satisfied if  $\lambda < 0$ . If  $\lambda > 0$ , collections and social welfare can be simultaneously increased by choosing an alternative taxation scheme which appropriately adjusts output. For output levels less than  $x_R$ ,  $B$  and  $R$  can be simultaneously increased by increasing output. Intuitively, both marginal social welfare ( $P-mc$ ) and marginal tax collections ( $mr - mc$ ) are positive in this range. Similarly, for output levels in excess of  $x_B$ ,  $B$  and  $R$  can be simultaneously increased by reducing output. Any taxation scheme which moves the industry into these regions is suboptimal. One can collect more and simultaneously raise social welfare by moving the industry to an output level between  $x_R$  and  $x_B$ . Essentially, the industry should not operate on the wrong-side of the collections mound (i.e., where  $x < x_R$ ) or on the wrong-side of the social

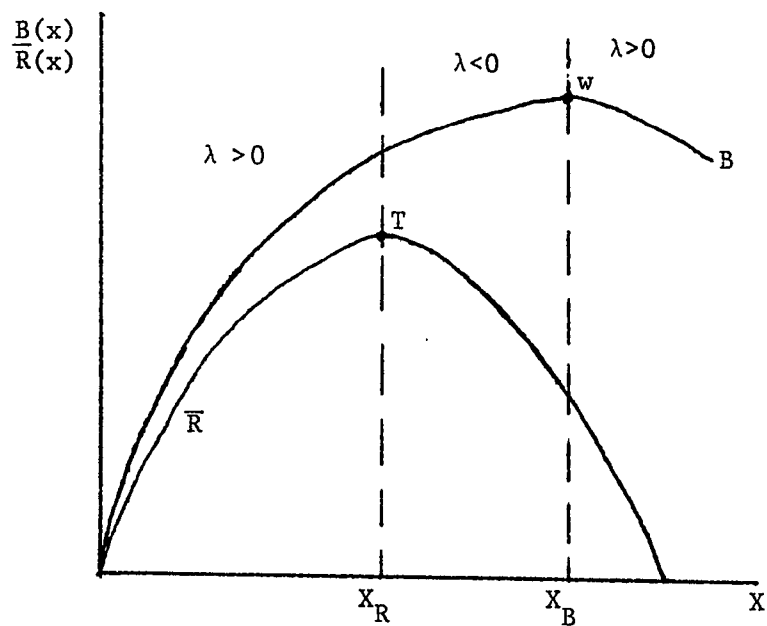


Figure II-1. The tax collection and social welfare functions

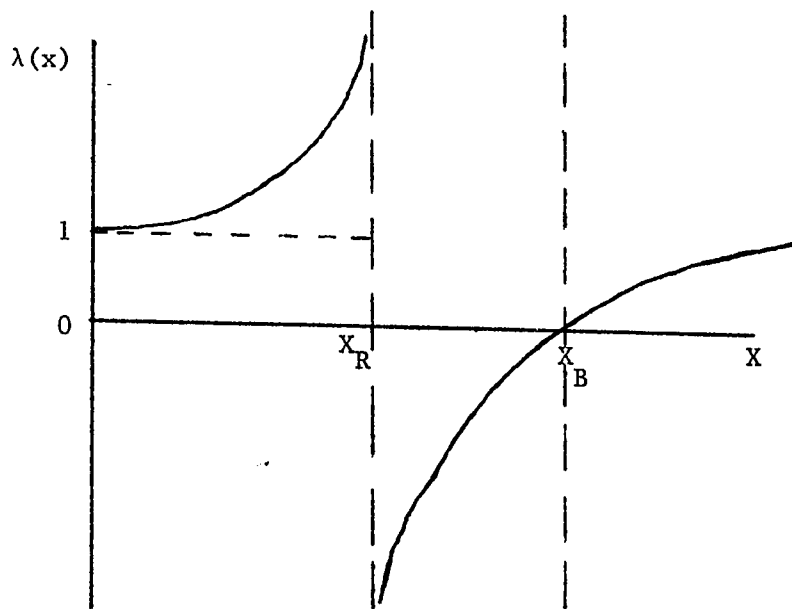


Figure II-2. The Lagrangean multiplier

welfare mound (i.e., where  $x > x_B$ ) such that  $\lambda > 0$  and the potential for pareto moves exists. Therefore,  $\lambda$  must be negative, implying that  $P > mc$  and  $mc > mr$  which illustrates the tradeoff problem inherent with optimal taxation. In order to optimally increase collections, social welfare must be reduced.

Figure II-2 explicitly illustrates the relationship between  $\lambda$  and output using equation (II-15). When output is zero,  $P = mr$ , and  $\lambda = 1$ . As output is increased,  $\lambda$  increases towards  $+\infty$  at the peak of the collections mound where  $mr = mc$ . For a marginal increment in  $x$  in excess of  $x_R$ ,  $\lambda \rightarrow -\infty$ , since  $mc > mr$ , while  $P > mc$ . It remains negative and increases to zero when social welfare is maximized (i.e.,  $P = mc$ ). Finally,  $\lambda$  is again positive for any output levels in excess of  $x_B$ . Therefore, the Lagrangean multiplier changes sign at the output level where the constraint is maximized and at the output level where the objective function is maximized. However, optimal taxation schemes move each industry only in the range where  $\lambda < 0$ .

## CHAPTER III. PERFECT COMPETITION

This chapter characterizes the optimal taxation path for a single competitive industry and determines that tax or combination of taxes required to move the industry along its optimal path. In addition, those taxes which are required to maximize collections from the industry are examined.

In the next section, a conventional model describing a perfectly competitive industry is developed. This model is used to describe the impact that the various taxes have on the competitive long-run equilibrium. Next, the optimal taxation path is derived and its characteristics are examined. Optimal taxation schemes are then derived and it is shown that a single excise tax is all that is required for optimal and maximal taxation. Finally, the results are extended to the general equilibrium framework discussed in the last chapter.

## Taxation and Perfect Competition

Imagine a perfectly competitive, constant-cost industry consisting of  $n$  identical firms producing a homogeneous product,  $q$ . Each firm incurs production cost equal to  $c(q)$  and faces a conventional u-shaped average cost structure, i.e.,  $AC = \frac{c(q)}{q}$ . Let  $P(Q^d)$  (where  $P'(Q^d) < 0$ ) represent the industry's inverse demand function, where  $Q^d$  is the industry quantity demanded and  $Q=nq$  is total industry output. Each firm maximizes profits by equating price to marginal cost ( $MC = c'(q)$ ) while free exit and entry ensure that long-run economic profits ( $\pi$ ) are zero. Finally, the long-run equilibrium price,  $P(Q)$ , is determined by the intersection of industry demand and supply (where  $Q^d=Q$ ). It can be shown that unit and *ad valorem*

excise taxes have equivalent effects when levied in perfectly competitive markets. Therefore, analysis is simplified by considering only unit excise taxes and license fees. Given these assumptions, the long-run tax-inclusive competitive equilibrium can be represented by two equations in two unknowns ( $Q$  and  $n$ ) or

$$(III-1) \quad \pi = P(Q)q - c(q) - tq - L = 0$$

$$\text{and } (III-2) \quad \pi' = P(Q) - MC(q) - t = 0 \quad \text{where } q = \frac{Q}{n}.$$

Equation (III-1) stipulates that economic profit is zero in the long-run and equation (III-2) states that profits are maximized. The per-unit excise rate ( $t$ ) increases both marginal and average cost while the per-firm license fee ( $L$ ) increases the fixed cost of each firm.

In the absence of taxation, the long-run competitive equilibrium occurs where  $P = MC = AC$ , or when the particular  $Q, n$  combination simultaneously satisfies

$$(III-3) \quad P = MC$$

$$\text{and } (III-4) \quad MC = AC.$$

Therefore, these two equations define two loci whose intersection represents the market equilibrium in the absence of taxation. Total differentiation of equations (III-3) and (III-4) yields

$$(III-3a) \quad \left. \frac{dQ}{dn} \right|_{P=MC} = \frac{-c''q}{P'n - c''} > 0$$

$$\text{and } (III-4a) \quad \left. \frac{dQ}{dn} \right|_{MC=AC} = q, \text{ respectively.}$$

Both loci are positively-sloped in  $Q, n$  space as illustrated in Figure III-1.

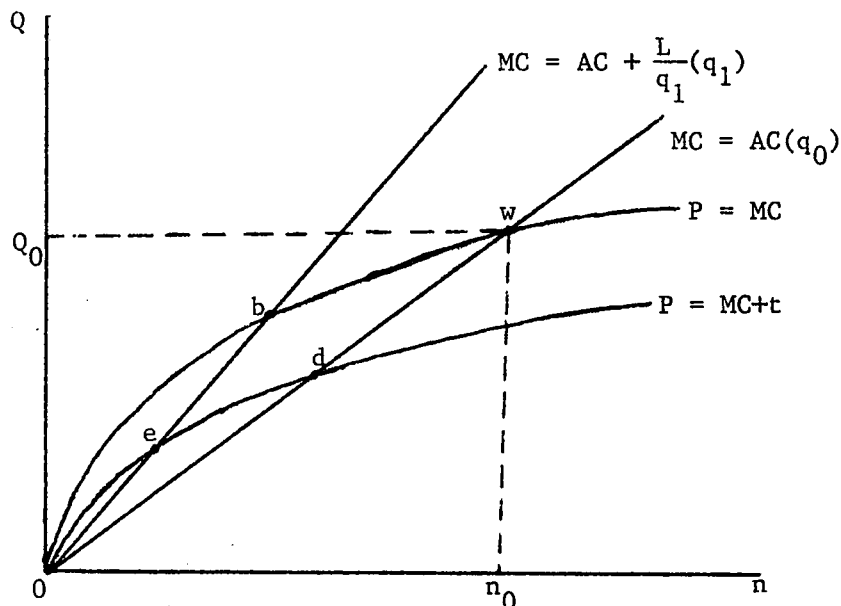


Figure III-1. Industry adjustment to license fees and excise taxes

Any ray originating from the origin has a slope equal to  $q$ . One such ray represents the  $MC=AC$  locus whose slope equals the output level produced by each firm in the absence of taxation ( $q_0$ ). Points above (below) this locus represent larger (smaller) per-firm output levels where  $MC > AC$  ( $MC < AC$ ). From equation (III-3a), any ray from the origin must cut the  $P=MC$  locus from below, which implies that it is concave from below. Similarly, points above (below) the  $P=MC$  locus represent  $Q, n$  combinations where  $P > MC$  ( $P < MC$ ). The initial long-run competitive equilibrium is, therefore, represented by point  $w$ . What then is the impact of excise and license taxation on this equilibrium?

From equations (III-1) and (III-2), the tax-inclusive long-run

equilibrium must simultaneously satisfy<sup>1</sup>

$$(III-5) \quad P = MC+t \quad \text{and} \quad (III-6) \quad MC = AC + \frac{L}{q} .$$

Equations (III-5) and (III-6) define two tax-inclusive loci corresponding to the loci defined by equations (III-3) and (III-4). Essentially, the  $P=MC$  and  $MC=AC$  loci represent a special case of the tax inclusive loci (i.e., when  $t=L=0$ ). Taxation simply shifts these loci and their new intersection determines the new tax-inclusive long-run equilibrium. First, consider excise taxation (i.e.,  $t>0$ ,  $L=0$ ). From (III-6), the new equilibrium must lie on the  $MC=AC$  locus. That is, excise taxes do not affect output per-firm. However, from (III-5), an excise tax pivots the  $P=MC$  locus inward to  $P=MC+t$ , so that a new equilibrium is established at point d where both  $Q$  and  $n$  are reduced. Consequently, as  $t$  is increased over the full range of potential rates, the industry adjusts along the  $MC=AC$  locus from point w to point 0. Similarly, excise subsidies move the industry to points northwest of point w along the  $MC=AC$  locus. The most notable characteristic of excise taxes is that they do not distort firms' production efficiency. Those firms that remain in the new long-run equilibrium continue to produce at minimum per-unit cost (i.e., where  $MC=AC$ ). Next, consider license fees (i.e.,  $L>0$ ,  $t=0$ ). A license fee is perceived as an addition to fixed cost and, therefore, increases average cost (to  $AC + \frac{L}{q}$ ), but does not affect marginal cost. The  $P=MC$  locus is unaffected by the license fee, while the  $MC=AC$  locus pivots upward to  $MC=AC + \frac{L}{q_1}$  as  $L$  is increased. The new license-inclusive equilibrium is illustrated by point b which represents a larger output per-firm ( $q_1$ ), a lower sales level, and a smaller

<sup>1</sup>Equation (III-6) is obtained by dividing equation (III-1) by  $q$ , then substituting equation (III-5) for  $P$  into equation (III-1).

number of firms. Essentially, as  $L$  is increased, output per-firm and, thus,  $MC$  must increase, until  $MC$  equals the (now larger) license-inclusive average cost. License fees, therefore, move the industry along the  $P=MC$  locus to points southwest of point  $w$ , while license subsidies move the industry to points northeast of point  $w$ . In contrast to excise taxes, licenses encourage firms to expand production to inefficiently large output levels.

Finally, the adjustment of the industry to simultaneous use of both taxes can be illustrated. Once a license fee is levied, an excise tax can only move the industry along the  $MC=AC + \frac{L}{q_1}$  locus. Similarly, if an excise tax is levied, licenses can only move the industry along the  $P=MC+t$  locus. Use of either tax, therefore, defines the locus along which the industry can be moved by the other tax instrument. As a specific example, consider a dual licensing-excise scheme. An individual firm is represented in Figure III-2.

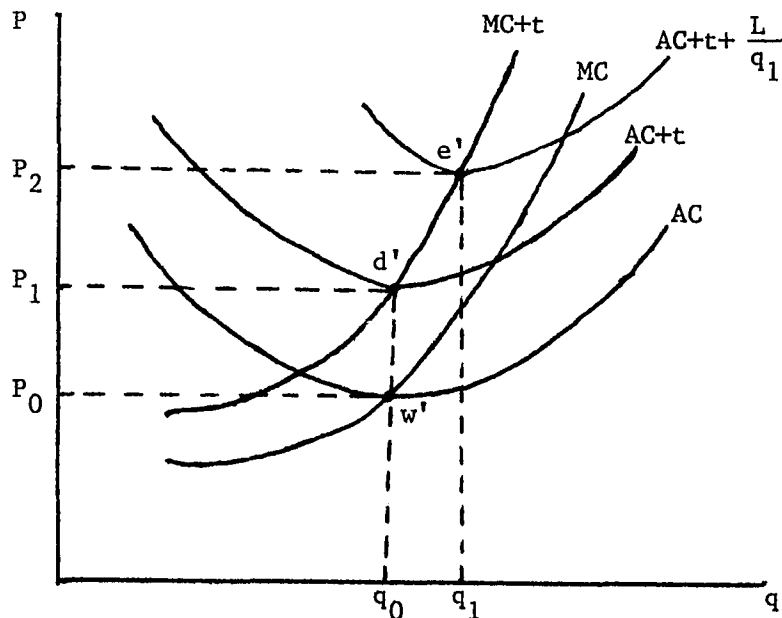


Figure III-2. The impact of a dual licensing-excise scheme



Point  $w$  (Figure III-1) and  $w'$  (Figure III-2) represent the initial long-run equilibrium in the absence of taxation. That is,  $n_0$  firms produce  $q_0$  units at minimum per-unit cost. A unit excise tax,  $t$ , shifts both the AC and MC curves upward to  $AC+t$  and  $MC+t$ , respectively. The new long-run equilibrium occurs at point  $d'$ . Remaining firms continue to produce  $q_0$  units and price rises by the full amount of the excise (to  $P_1$ ). The corresponding industry adjustment is illustrated in Figure III-1 by the movement from point  $w$  to point  $d'$  along the  $MC=AC$  locus. If a license fee is also levied, the average cost curve is shifted up along the marginal cost schedule to  $AC + t + \frac{L}{q_1}$ . Remaining firms minimize per-unit cost by expanding output to  $q_1$  where a new equilibrium is established at point  $e'$ . The corresponding industry adjustment is represented by the move from point  $d$  to point  $e$  along the  $P=MC+t$  locus in Figure III-1.

#### The Optimal Taxation Path

We now proceed to characterize the optimal taxation path for a perfectly competitive industry. The problem is to determine that taxation scheme which maximizes social welfare from the industry given any required collections level.

In the last chapter (see equation II-9a), it was shown that the consumer surplus or total social welfare attributable to the consumption of this industry's good is

$$(III-7) \quad B = \int_0^Q P(k) dk - nc \left( \frac{Q}{n} \right),$$

assuming any tax collections are rebated in a lump-sum fashion. The first term is the area under the demand schedule (i.e., gross consumer benefits)

and the second term represents total industry cost (i.e., the number of firms times the cost per-firm). Welfare ( $B$ ) is defined in terms of two variables,  $Q$  and  $n$ . Assigning a specific value to  $B$  (e.g.,  $B_0$ ) defines an iso-welfare surface which consists of the locus of  $Q, n$  combinations, such that  $B = B_0$ . Setting the total differential of equation (III-7) equal to zero yields the slope of this surface or

$$(III-7a) \quad \left. \frac{dQ}{dn} \right|_{\bar{B}} = \frac{q(AC-MC)}{P-MC} .$$

One such iso-welfare surface ( $B = B_0$ ) is illustrated in Figure III-3 where the  $P=MC$  and  $MC=AC$  loci discussed in the last section are also reproduced.

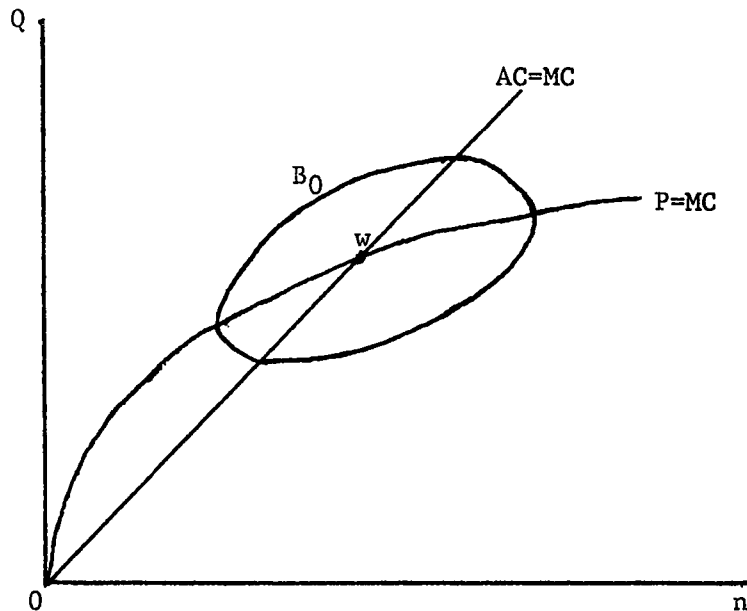


Figure III-3. An iso-welfare surface

From equation (III-7a), the slope of an iso-welfare surface is zero along the  $AC = MC$  locus and infinity along the  $P = MC$  locus. Therefore, each surface is represented by a quasi-ellipse. Point  $w$  illustrates the unconstrained welfare optimum  $Q, n$  combination. Such an optimum satisfies the first order conditions for a maximum of  $B$ , or from equation (III-7)

$$(III-8) \quad \frac{\partial B}{\partial Q} = P - MC = 0$$

$$\text{and } (III-9) \quad \frac{\partial B}{\partial n} = q(AC - MC) = 0.$$

Higher (lower) levels of welfare are represented by surfaces which lie closer to (farther from) point  $W$ . The problem then is to move the industry (via taxation) onto the highest welfare surface consistent with any given collections constraint,  $R_0$ .

Total collections consist of the sum of excise and license collections. That is,

$$(III-10) \quad R = tQ + nL.$$

Since there is free exit and entry in the long-run, the net supply price is always driven to equality with net per-unit cost. Consequently, collections can also be expressed as the difference between the gross demand and net supply prices times the industry sales level. That is,

$$(III-11) \quad R = P(Q)Q - nc \left( \frac{Q}{n} \right),$$

which is equal to the gross-of-tax industry profit. A given collections constraint shows all  $Q, n$  combinations which yield the same level of collections (say  $R = R_0$ ). Its slope is found by setting the total differential of equation (III-11) equal to zero, or

$$(III-11a) \quad \left. \frac{dQ}{dn} \right|_{\bar{R}} = \frac{q(AC-MC)}{MR-MC}$$

where MR is the gross industry marginal revenue ( $P + P'Q$ ). From equation (III-11), collections are maximized when

$$(III-12) \quad \frac{\partial R}{\partial Q} = P + P'Q - MC = MR - MC = 0$$

$$\text{and } (III-13) \quad \frac{\partial R}{\partial n} = q(AC-MC) = 0.$$

Equation (III-13) is satisfied by any  $Q, n$  combination on the  $AC=MC$  locus. Total differentiation of equation (III-12) yields the slope of the  $MR=MC$  locus, or

$$(III-12a) \quad \left. \frac{\partial Q}{\partial n} \right|_{MR=MC} = \frac{-c''q}{(2P' + P''Q)n - c''} > 0 \quad \cdot$$

This locus is positively-sloped and must lie below the  $P=MC$  locus, since  $P > MR$  at any positive  $Q, n$  combination. Similar to the  $P=MC$  locus, the  $MR=MC$  locus is also concave from below. That is, equation (III-12a) shows that any ray originating from the origin has a greater slope (equal to  $q$ ) than the  $MR=MC$  locus at the point of their intersection. These three loci and a given collections constraint ( $R=R_0$ ) are illustrated in Figure III-4. Examination of equation (III-11a) shows that a particular collections constraint is represented by a quasi-ellipse similar to an iso-welfare surface. Its slope is zero along the  $MC=AC$  locus and infinity along the  $MR=MC$  locus. Point T represents the  $Q, n$  combination which maximizes collections. Note that collections are maximized at a  $Q, n$  combination such that industry MR equals industry MC. This is exactly the  $Q, n$  combination which a multi-

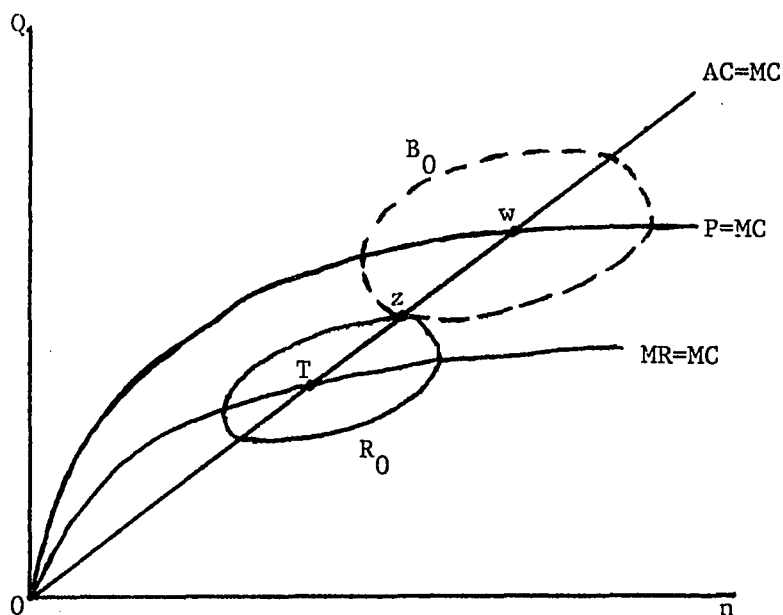


Figure III-4. The optimal taxation path

plant monopolist would choose in the absence of taxation. That is, the taxing authority essentially induces the industry to act as would a monopolist when it desires to maximize collections. Similar to iso-welfare surfaces, higher (lower) collection levels are represented by constraints which lie closer to (farther from) point T.

The optimal taxation path can now be derived. For any given collections constraint, the industry must be moved to the highest attainable iso-welfare surface consistent with the constraint. One such optimal taxation point is represented by point z in Figure III-4. For the collections level  $R_0$ , the highest attainable welfare level is  $B_0$ , since at point z the iso-welfare surface is tangent to the collections constraint. The entire optimal taxation path can be derived by changing the collections constraint and finding a new point of tangency to the highest obtainable iso-welfare

surface. Formally, a point on the optimal taxation path is defined by the  $Q, n$  combination which maximizes

$$(III-14) \quad \mathcal{L} = \int_0^Q P(k) dk - nc\left(\frac{Q}{n}\right) + \lambda \left[ R_0 - P(Q)Q + nc\left(\frac{Q}{n}\right) \right].$$

All such points can be determined by changing the value of  $R$  and re-maximizing. Thus, each  $Q, n$  combination on the optimal taxation path must simultaneously satisfy

$$(III-14a) \quad \frac{\partial \mathcal{L}}{\partial Q} = P - MC - \lambda(MR - MC) = 0$$

$$\text{and } (III-14b) \quad \frac{\partial \mathcal{L}}{\partial n} = q(AC - MC)(1 + \lambda) = 0$$

$$\text{and } (III-14c) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = R_0 - P(Q)Q + nc\left(\frac{Q}{n}\right) = 0$$

for the corresponding collections level. From equation (III-14a),  $\lambda = \frac{P - MC}{MR - MC}$ . Therefore,  $\lambda$  cannot equal  $-1$ , since that would require  $P = MR$  which only occurs if  $Q = 0$ . Consequently, given  $\lambda \neq -1$ , equation (III-14b) is only satisfied when  $AC = MC$ . That is, the optimal taxation path consists of all  $Q, n$  combinations which lie on the  $AC = MC$  locus on or to the north-east of point  $T$ . This can also be shown by examining Figure III-4. Both the iso-welfare surface and the collecting constraint have a zero slope at all points on the  $AC = MC$  locus. Therefore, they are tangent at all points along this locus. Note that points on this locus between point  $O$  and point  $T$  cannot be optimal. At any point between points  $O$  and  $T$ , a higher iso-welfare surface for the same collections level can be obtained by moving the industry to a point between points  $T$  and  $w$ .

In general, optimal taxation in perfectly competitive markets requires that output per-firm remain unaltered. As collections are increased, both the level of industry sales and the number of firms must be reduced by an equal proportion so that firms maintain their productive efficiency.

#### Optimal Taxation Schemes

We can now determine which of the taxation schemes discussed earlier meet the requirements of an optimal taxation scheme. Recall that excise taxes move the industry along the  $AC=MC$  locus, while license fees result in industry adjustment along the  $P=MC$  locus. Therefore, the industry adjustment path under excise taxation coincides with the optimal path. This is illustrated in Figure III-5. The segment  $WT$  of the excise taxation path coincides with the optimal taxation path. However, license fees move the industry off the  $AC=MC$  locus and are, therefore, suboptimal tax instruments in perfectly competitive industries. Due to their equivalent effects, either unit or *ad valorem* excise taxes can move the industry along the optimal path. Essentially, optimal taxation requires that firms' production efficiency not be distorted. While license fees induce firms to expand production and incur higher per-unit cost, excise taxes maintain production efficiency at minimum average cost. Therefore, a single excise tax is all that is required for optimal taxation and licenses are inappropriate in perfectly competitive industries.

Not only are licenses inefficient, but their revenue potential is also less than excise taxes. The collections functions for excise ( $R_t$ ) and license taxation ( $R_L$ ) are illustrated in Figure III-6. In the absence of taxation, the industry operates at point  $w$  where  $P=MC$  and profits are

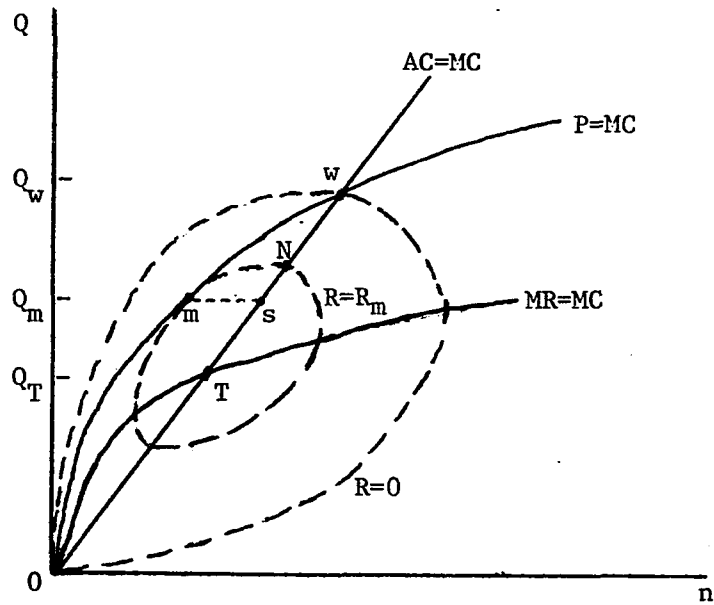


Figure III-5. Optimal taxation schemes

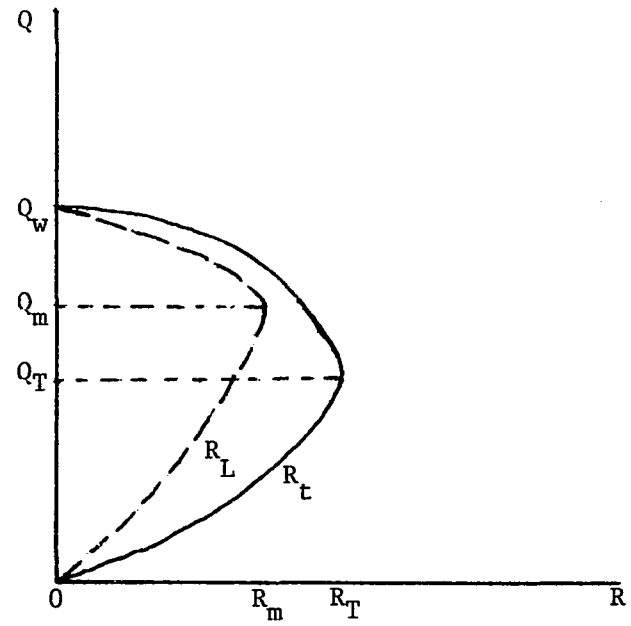


Figure III-6. License and excise collection functions



zero (i.e.,  $R = 0$ ). Note that the  $R=0$  locus must go through the origin. When both  $Q$  and  $n$  are zero, there is no tax base and collections must necessarily equal zero. Excise taxes move the industry along the  $AC=MC$  locus and collections increase from zero at point  $w$  to a maximum at point  $T$ . Excise taxation beyond point  $T$  moves the industry into the "prohibitive range" along its collections function. As license fees are increased, the industry moves away from point  $w$  along the  $P=MC$  locus. Collections are increased from zero at point  $w$  to a maximum at point  $M$  where the collections constraint,  $R_m$ , is just tangent to the  $P=MC$  locus. For larger license fees, the industry is moved to the southwest of point  $M$  and collections are reduced. As Figure III-6 illustrates, for any industry sales level, excise collections always exceed license collections. For example, when  $Q=Q_m$ , license collections equal  $R_m$ , which is less than the excise collections constraint going through point  $s$  (not drawn) at the same sales level. Intuitively, licenses induce firms to adopt inefficient production levels and, therefore, part of potential tax collections (realized by excise taxes) is forfeited to higher production cost.

In general, excise taxation is required for optimal taxation of perfectly competitive industries. While multiple excise schemes can move the industry along its optimal path, the simplest method is a single excise tax. Moreover, licenses generally should not be used. They move the industry away from its optimal path and have a smaller revenue potential than excise taxes.

Most analyses concerned with optimal commodity taxation have been limited to taxation of perfectly competitive markets. Because of the

superiority of excise taxation, the major issue has been whether a uniform or a differential "excise" rate structure among industries is required. Therefore, there has been very little analysis of whether other tax instruments or a multiple taxation scheme may be required. It will be shown in ensuing chapters that this superiority of excise taxation is not a general result, but, rather, is specific to perfectly competitive industries. When one allows for imperfect competition, optimal taxation generally requires a wider range of tax instruments and the use of multiple taxation schemes.

#### The General Equilibrium Problem

In this chapter, a partial equilibrium approach was used to derive the optimal taxation scheme for a single competitive industry. How do the results of this chapter relate to the general equilibrium problem discussed in Chapter II?

From equation (III-14a), it was shown that optimal taxation requires  $\lambda = \frac{P-MC}{MR-MC}$ . The value of  $\lambda$  at all points along this industry's optimal taxation path (i.e., for every potential value of  $R$ ) can be illustrated by examining Figure III-5.  $\lambda$  is zero at point  $w$  (since  $P=MC$ ) and decreases continually along the segment  $WT$  approaching  $-\infty$  as point  $T$  is reached and collections from the industry are maximized (i.e., when  $MR=MC$ ). Recall that optimal taxation of the economy requires equality of  $\lambda$ s for all industries simultaneously. In this chapter, we have derived that tax scheme which moves this particular industry along its optimal path. That is, we determined a tax scheme which can yield any value for  $\lambda$ . Once this is completed for all industries, the general equilibrium optimal taxation problem

is easily solved by simply using these appropriate tax schemes, such that  $\lambda$  is equated across all industries. Consequently, while a partial equilibrium approach is used, it provides the necessary information to optimally tax the entire economy.

## CHAPTER IV. MONOPOLISTIC COMPETITION

This chapter investigates the optimal taxation problem when the taxing authority enters a monopolistically competitive industry. The characteristics of monopolistic competition differ greatly from those of perfectly competitive markets. While perfectly competitive markets operate pareto efficiently in the absence of taxation, price exceeds marginal cost under monopolistic competition. In addition, monopolistically competitive firms each produce a distinct product and, therefore, consumers' valuations of their output depend on the level of variety (i.e., number of firms), in addition to the total quantity consumed. Finally, each firm faces a downward-sloping demand schedule which results in production above minimum average cost in the long-run. Because of these differences, characteristics of the optimal taxation path and optimal taxation schemes differ from those derived in perfectly competitive markets.

In his famous work on monopolistic competition, Chamberlin [8] distinguished between two types of competitive behavior --that is, industries characterized with and without active price competition. Both types of competitive behavior are discussed in the next section. A mathematical model is developed and the long-run industry equilibrium conditions for both types of competitive behavior are derived. Next, characteristics of the optimal taxation path for any monopolistically competitive industry are examined. While the exact path depends on the specific values of the parameters in the model, the region within which the optimal path generally lies is determined. Optimal taxation schemes are analyzed in the following section, both when the industry is characterized by price and

nonprice competition. Contrary to perfect competition, optimal taxation generally requires a combination of taxes. Finally, the last section gives an intuitive rationale for these results.

#### Price and Nonprice Competition

This section discusses the basic model which defines the long-run industry equilibrium position when the industry is characterized by either price or nonprice competitive behavior. The system of equations describing these equilibria will be used throughout the remaining sections of this chapter.

Each firm in a monopolistically competitive industry produces a differentiated product. Therefore, industry demand is a function of the level of variety available in addition to the quantity consumed. Let the industry's inverse demand function be

$$(IV-1) \quad P = P(Q, n)$$

where  $Q = nq$  is total industry output (assumed to be equal to the quantity demanded in the long-run equilibrium),  $n$  represents the number of firms (i.e., the level of variety) existing in the industry, and  $q$  is output per firm. The industry demand price is inversely related to  $Q$ , and  $n$  has a positive, but diminishing, impact on  $P$ . That is, assume  $P_Q < 0$ ,  $P_{QQ} \leq 0$ ,  $P_n > 0$ , and  $P_{nn} < 0$ . Finally, assume that the responsiveness of  $P$  to changes in  $Q$  diminishes as  $n$  increases, or  $P_{Qn} > 0$ . A specific demand function which displays these properties is

$$(IV-2) \quad P = A - Q \left[ b + \frac{a-b}{n} \right] \text{ where } A, a, \text{ and } b > 0 \text{ and } a > b.$$

This describes a linear demand function in P,Q space which pivots upward about its vertical intercept (A) as n is increased.

Since all firms are assumed to face identical demand and cost functions, a single industry price exists at all times. However, individual firms do not respond to equation (IV-2), but, rather, base output decisions on what they perceive to be the demand function they face. Equation (IV-2) can be manipulated to express the price faced by the  $i^{\text{th}}$  firm ( $P_i$ ) as a function of its output and the remaining firms' output levels, or

$$(IV-3) \quad P_i = A - aq_i - b \sum_{\substack{j=1 \\ j \neq i}}^{n-1} q_j .$$

If the industry is characterized by price competition, then each firm  $i$

assumes  $\frac{\partial q_i}{\partial q_j} = 0$ , and, therefore, the slope of its "perceived" demand func-

tion is  $\frac{\partial P_i}{\partial q_i} = -a$ . However, since all firms' output levels move together,

each actually faces the following "proportional" demand function (derived from equation (IV-3) by letting  $q_i = q_j$ ),

$$(IV-4) \quad P_i = A - q_i [b(n-1)+a] .$$

Inspection of equations (IV-3) and (IV-4) show that the absolute slope of the proportional demand function is greater than that of the perceived demand function. This results in active price competition. Each firm believes it can lower price and increase sales without reactions by rivals. That is, all believe they can expand their sales with a small reduction in price by moving down their perceived demand functions. However, since all

firms expand output simultaneously, all actually move down their true or proportional demand schedule (i.e., the perceived demand curve slides along the proportional demand schedule) and realize a smaller increase in sales at a lower price than expected *a priori*.

Active price competition breaks down when firms correctly realize their mutual interdependence. That is, nonprice competition exists when each firm correctly realizes that it faces the proportional demand function. In this case, firms realize that price cutting is met by rivals so that all are worse off as they are forced to move down their proportional demand schedules. A "live and let live" outlook, tacit agreements, and/or open price associations may result and lead to the absence of active price competition. Despite this behavior, entry or exit occurs until economic profits are zero in the long-run. Therefore, the perceived demand function plays no role and nonprice competitors base output decisions only on their proportional demand function.

The slopes of the industry, proportional, and perceived demand functions can be derived from equations (IV-2), (IV-4), and (IV-3), respectively. That is,

$$(IV-2a) \quad \frac{\partial P}{\partial Q} = - \left[ b + \frac{a-b}{n} \right] = P_Q, \text{ for the industry demand,}$$

and

$$(IV-4a) \quad \frac{\partial P_i}{\partial q_i} = - \left[ b(n-1)+a \right] = -n \left[ b + \frac{a-b}{n} \right] = P_{Qn}, \text{ for the proportional demand,}$$

and

$$(IV-3a) \quad \frac{\partial P_i}{\partial q_i} = -a = P_q, \text{ for the perceived demand.}$$

Therefore,  $P_{Qn} < P_q < P_Q < 0$ .

We will assume that each firm faces a falling long-run average cost structure (AC) of the form<sup>1</sup>

$$(IV-5) \quad AC = c + \frac{F}{q},$$

where  $c$  represents a constant marginal cost and  $F$  is total fixed cost per firm. Free entry in the long-run ensures long-run economic profits ( $\pi$ ) are zero, or

$$(IV-6) \quad \pi = P(Q,n)q - cq - F = 0,$$

and each firm maximizes its profit by equating its perceived marginal revenue to marginal cost. That is

$$(IV-6a) \quad \pi' = P + P_Q Q - c = 0 \quad \text{for nonprice competition,}$$

$$\text{and } (IV-6b) \quad \pi' = P + P_q q - c = 0 \quad \text{for price competition.}$$

The long-run industry equilibrium is then defined by equations (IV-6) and (IV-6a) for nonprice competition and by equations (IV-6) and (IV-6b) when the industry is characterized by price competition. These two equilibria are illustrated in Figure IV-1. The price competitive long-run equilibrium is represented by point  $E_p$ , where the perceived demand schedule ( $dd$ ) intersects the proportional demand ( $DD_p$ ) and is tangent to AC. When nonprice competition characterizes the industry, long-run equilibrium occurs at point  $E_N$  where the proportional demand schedule ( $DD_N$ ) is tangent to AC. As Figure IV-1 illustrates, price competitors produce a larger output at a lower per-unit cost in the long-run.

---

<sup>1</sup>Since all the relevant equilibria occur where firms' average cost is falling, the results are not significantly changed by assuming instead a U-shaped average cost schedule.



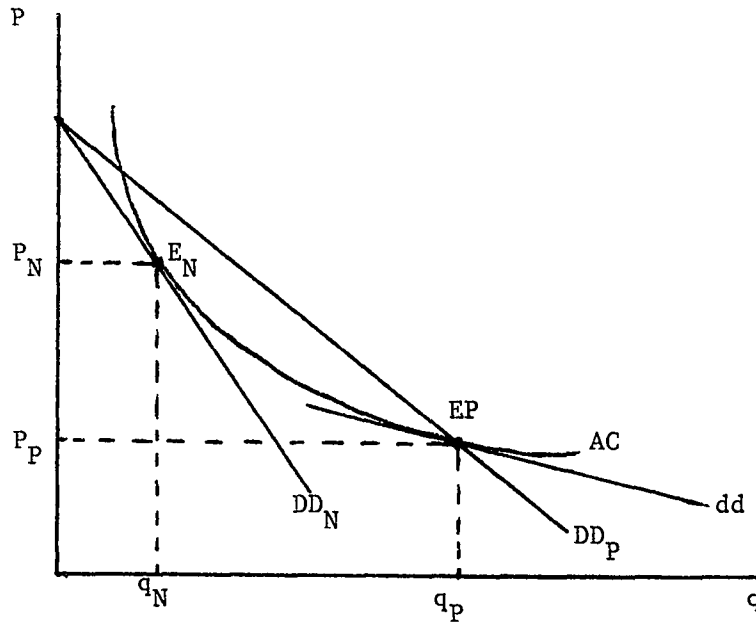


Figure IV-1. Price and nonprice equilibria

Optimal taxation schemes will be analyzed for both types of market equilibria. Our next task, however, is to derive the optimal taxation path for a monopolistically competitive industry irrespective of the type of competitive behavior which exists in the industry.

#### The Optimal Taxation Path

Let social welfare from the industry (with taxes rebated in a lump-sum fashion) be defined by

$$(IV-7) \quad B = \int_0^Q P(k,n)dk - cQ - nF,$$

which consists of gross consumer benefits (which now depend on the level of variety,  $n$ , in addition to the quantity consumed) minus total industry cost. Since economic profits are dissipated in the long-run by free entry,

tax collections are equal to gross (of tax) industry profits, or

$$(IV-8) \quad R = P(Q,n)Q - cQ - nF.$$

The optimal taxation path is defined by the locus of  $Q,n$  combinations which maximize  $B$  for all potential values of  $R$ . Formally, it is represented by moving the industry to the highest attainable iso-welfare surface consistent with any given collections constraint.

First, consider the collections constraint. From equation (IV-8), collections are maximized when

$$(IV-9) \quad \frac{\partial R}{\partial Q} = P + P_Q Q - c = MR(Q) - MC(Q) = 0$$

$$\text{and } (IV-10) \quad \frac{\partial R}{\partial n} = P_n Q - F = MR(n) - MC(n) = 0$$

are simultaneously satisfied. These equations define two loci whose intersection represents the  $Q,n$  combination required for maximal collections. Setting the total differential of equation (IV-10) equal to zero yields the slope of the  $MR(n) = MC(n)$  locus, or

$$(IV-10a) \quad \left. \frac{dn}{dQ} \right|_{MR(n) = MC(n)} = \frac{-(P_n + P_{nQ}Q)}{P_{nn}Q} > 0.$$

This locus represents all  $Q,n$  combinations, such that the marginal revenue of another firm ( $MR(n) = P_n Q$ ) equals its marginal cost ( $MC(n) = F$ ). The slope of this locus is positive, and with the specific demand function defined by equation (IV-2), its slope can be shown to equal  $1/q$ . This locus is illustrated in Figure IV-2 and is simply a ray originating from the origin. That is, there is only one output level per-firm, such that  $MR(n) = MC(n)$ .

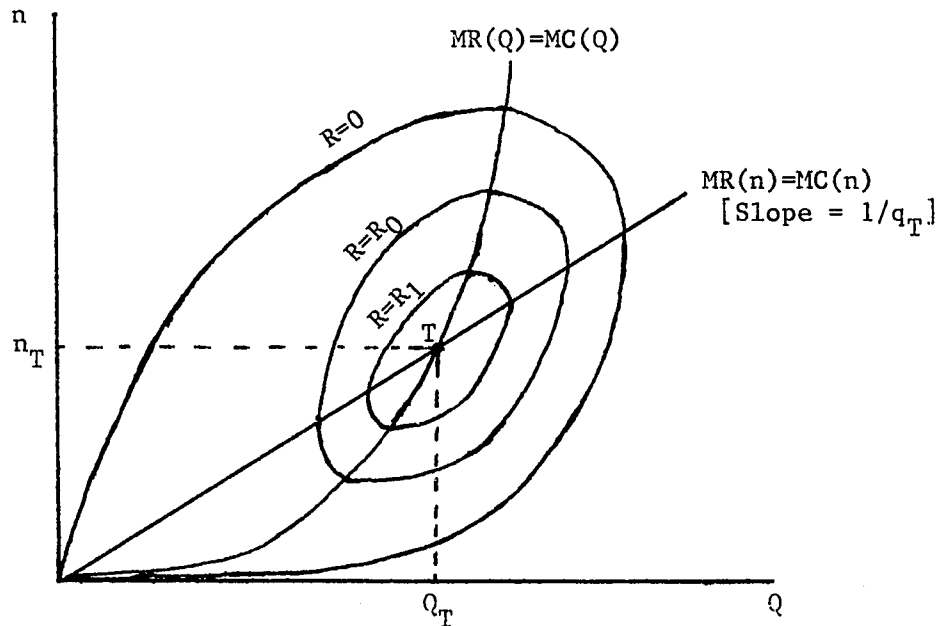


Figure IV-2. Collection constraints

Next, consider the  $MR(Q) = MC(Q)$  locus, which represents all  $Q, n$  combinations such that the marginal revenue of an additional  $Q$  ( $MR(Q) = P + P_Q Q$ ) equals its marginal cost ( $MC(Q) = c$ ). From equations (IV-9) and (IV-2), its slope is found to be

$$(IV-9a) \quad \left. \frac{dn}{dQ} \right|_{MR(Q)=MC(Q)} = \frac{-\left(2P_Q + P_{QQ}Q\right)}{P_n + P_{nQ}Q} = \frac{b(n-1)+a}{(a-b)q} > 1/q,$$

which is positive and cuts any ray originating from the origin (whose slope equals  $1/q$ ) from below. Thus, the  $MR(Q)=MC(Q)$  locus originates from the origin and is concave from above, as illustrated in Figure IV-2. The point of maximal collections is represented by point T. Essentially, this is the point at which a multi-plant monopolist would operate so as to maximize its profits. Therefore, if the government were to maximize collections, it would induce the industry to act like a multi-plant monopolist. A given collections constraint is defined by equation (IV-8) by letting  $R = R_0$ .

The slope of this constraint is

$$(IV-11) \quad \left. \frac{dn}{dQ} \right|_{\bar{R}} = \frac{-(P + P_Q Q - c)}{P_{nQ} - F} = \frac{-[MR(Q) - MC(Q)]}{MR(n) - MC(n)} .$$

Note that  $MR(n) - MC(n) > 0$  ( $\leq 0$ ) at all  $Q, n$  combinations below (above or on) the  $MR(n) = MC(n)$  locus and  $MR(Q) - MC(Q) > 0$  ( $\leq 0$ ) at all  $Q, n$  combinations above (below or on) the  $MR(Q) = MC(Q)$  locus. Consequently, collection constraints are illustrated in Figure IV-2 as quasi-ellipses drawn around the  $(Q, n)$  point of maximal collections. Constraints which lie closer to point T represent higher collection levels (i.e.,  $R_1 > R_0 > R = 0$ ). Note that the  $R=0$  constraint originates from the origin. When  $Q$  and  $n$  are both zero, there is no tax base; thus, collections must equal zero. The problem, then, is to choose the point on each constraint which yields the highest level of social welfare from the industry. Thus, to complete the analysis, we need to examine the welfare surface.

From equation (IV-7), social welfare from the industry is maximized when

$$(IV-12) \quad \frac{\partial B}{\partial Q} = P - c = P - MC(Q) = 0$$

$$\text{and } (IV-13) \quad \frac{\partial B}{\partial n} = \int_0^Q P_n(k, n) dk - F = MB(n) - MC(n) = 0$$

are simultaneously satisfied. These equations define two loci whose intersection represents the  $Q, n$  combination which maximizes social welfare from the industry. The  $P = MC(Q)$  locus shows all  $Q, n$  combinations such that the marginal gross benefit ( $P$ ) of an additional  $Q$  equals its marginal cost ( $MC(Q) = c$ ). Similarly, the  $MB(n) = MC(n)$  locus shows all  $Q, n$  combinations

such that the marginal gross benefit  $MB(n) = \int_0^Q P_n(k,n)dk$  of an additional firm equals its marginal cost ( $MC(n) = F$ ). These loci are illustrated in Figure IV-3.

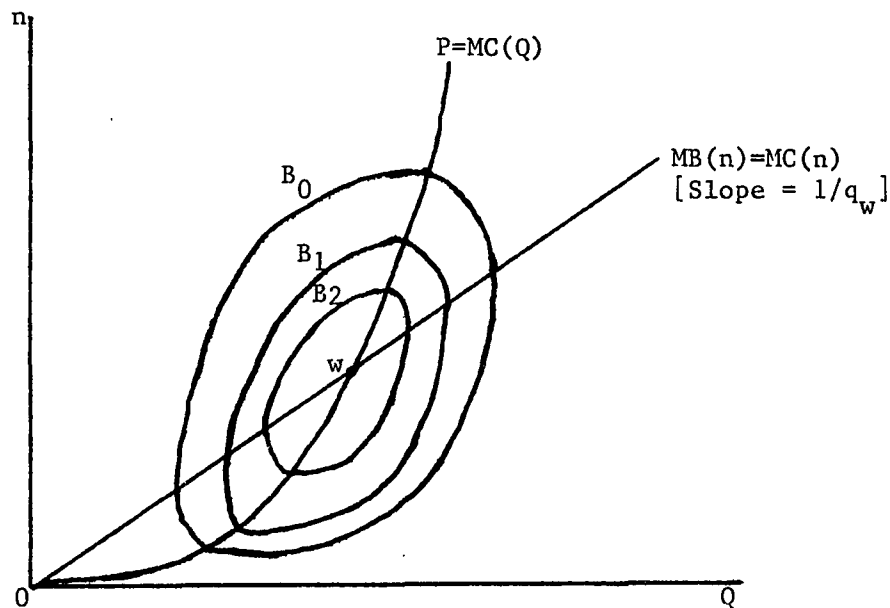


Figure IV-3. Iso-welfare surfaces

From equation (IV-13), the slope of the  $MB(n)=MC(n)$  loci can be shown to equal

$$(IV-13a) \quad \left. \frac{dn}{dQ} \right|_{MC(n)=MC(n)} = \frac{-P_n}{w_{nn}} > 0 \text{ where } w_{nn} = \int_0^Q P_{nn}(k,n)dk.$$

Using the specific demand function defined by equation (IV-2), its slope can be shown to equal  $1/q$ , which is simply a ray originating from the origin. Similarly using equation (IV-12), the slope of the  $P=MC(Q)$  loci is equal to

$$(IV-12b) \quad \left. \frac{dn}{dQ} \right|_{P=MC(Q)} = \frac{-P}{P} \frac{Q}{n} > 0 \quad \text{in general, and is equal to}$$

$$(IV-12b') \quad \left. \frac{dn}{dQ} \right|_{P=MC(Q)} = \frac{b(n-1)+a}{(a-b)q} > \frac{1}{q},$$

given the specific demand function defined by equation (IV-2). Therefore, any ray through the origin (whose slope equals  $1/q$ ) must cut the  $P=MC(Q)$  locus from above, which implies that the  $P=MC(Q)$  locus is concave from above. Welfare is maximized at the  $Q, n$  combination corresponding to point  $w$  where the two loci intersect and equations (IV-12) and (IV-13) are both satisfied. Iso-welfare surfaces are defined by all  $Q, n$  combinations which yield a given value of  $B$  (e.g.,  $B=B_0$ ) in equation (IV-7). From equation (IV-7), these surfaces have a slope equal to

$$(IV-14) \quad \left. \frac{dn}{dQ} \right|_{\bar{R}} = \frac{-(P-MC(Q))}{MB(n)-MC(n)}.$$

Therefore, iso-welfare surfaces are represented by quasi-ellipse (with slope of zero where  $P=MC(Q)$  and infinite slope where  $MB(n)=MC(n)$ ) drawn around point  $w$ , with successively higher welfare levels represented by surfaces which lie closer to point  $w$  (i.e.,  $B_2 > B_1 > B_0$ ).

We can now proceed to specify how the optimal taxation path is determined. For any given collections constraint, a point on the optimal path is represented by the highest attainable iso-welfare surface consistent with the constraint. That is, the point of tangency between the collections constraint and an iso-welfare surface; the optimal tax path is then a locus of such points. First, we need to determine where the loci in

Figure IV-2 lie relative to those in Figure IV-3. Consider the  $R=0$  collections locus in Figure IV-2. This locus consists of all  $Q, n$  combinations such that  $P = c + \frac{F}{q} = AC$ . Therefore, the  $P = MC(Q)$  (or  $P=c$ ) locus in Figure IV-3 must lie below the  $R=0$  locus since  $MC(Q) = c < AC(Q) = c + \frac{F}{q}$ . That is, at any point on the  $R=0$  locus,  $P=AC > c$ . Thus, at any given  $n$ ,  $Q$  must be increased to satisfy the  $P=c$  locus. Next, consider the  $MB(n)=MC(n)$  (or  $w_n=F$ ) and the  $MR(n)=MC(n)$  (or  $P_n Q=F$ ) loci. Utilizing equation (IV-2), it can be shown that

$$(IV-15) \quad w_n = \frac{(a-b)Q^2}{2n^2} \quad \text{and} \quad (IV-16) \quad P_n Q = \frac{(a-b)Q^2}{n^2}.$$

Note that, for any given  $Q$ , the  $n$  which equates  $P_n Q=F$  is greater than the  $n$  which satisfies  $w_n=F$ . Therefore, the  $w_n=F$  (or  $MB(n)=MC(n)$ ) locus must lie below the  $P_n Q=F$  (or  $MR(n)=MC(n)$ ) locus. These four loci are illustrated in Figure IV-4.

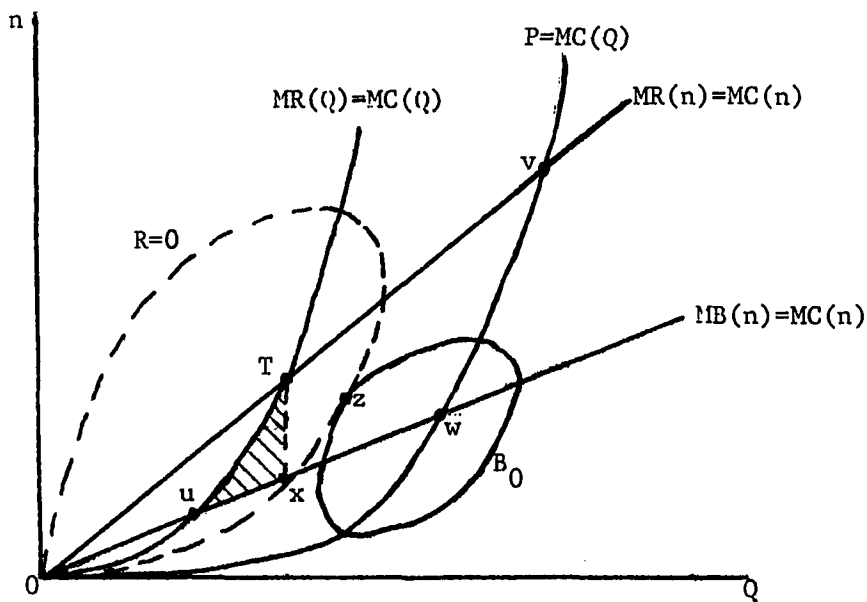


Figure IV-4. The optimal taxation path

Note that point  $w$  lies outside the zero collections constraint (i.e., the  $R=0$  locus). Intuitively, marginal cost pricing is required to maximize welfare. Since marginal cost is less than average cost, firms require a net subsidy (i.e.,  $R < 0$ ) to operate efficiently. If collections are constrained to zero, the optimal  $Q, n$  combination is that represented by point  $z$ . For the given level of collections ( $R=0$ ), point  $z$  represents the particular  $Q, n$  combination which moves the industry to the highest attainable iso-welfare surface ( $B_0$ ) consistent with the  $R=0$  collections constraint. The remaining  $Q, n$  combinations on the optimal taxation path can be determined by changing the collections constraint and finding points of tangency to the highest attainable iso-welfare surface. Formally, points on the optimal taxation path are found by maximizing  $B$  subject to  $R=\bar{R}$ . That is, the maximum of

$$(IV-17) \quad \mathcal{L} = \int_0^Q P(k, n) dk - cQ - nF + \lambda [\bar{R} - P(Q, n)Q + cQ + nF]$$

must satisfy

$$(IV-17a) \quad \frac{\partial \mathcal{L}}{\partial Q} = P - MC(Q) - \lambda (MR(Q) - MC(Q)) = 0,$$

$$(IV-17b) \quad \frac{\partial \mathcal{L}}{\partial n} = MB(n) - MC(n) - \lambda (MR(n) - MC(n)) = 0,$$

$$(IV-17c) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - P(Q, n)Q + cQ + nF = 0$$

simultaneously. Although the particular shape of the optimal taxation path depends on the specific model parameters, the region in which the optimal path can exist can be determined. First, assume  $\lambda < 0$ , so that the industry never operates on the pareto-inferior side of the tax collections (Laffer)



function or on the pareto-inferior side of the welfare function. Then, equation (IV-17a) is only satisfied if  $P-MC(Q)$  and  $MR(Q)-MC(Q)$  are of opposite signs. Points below the  $P=MC(Q)$  locus represent  $Q, n$  combinations such that  $P-MC(Q)$  and  $MR(Q)-MC(Q)$  are both negative. Similarly, at all points above the  $MR(Q)=MC(Q)$  locus, both  $P-MC(Q)$  and  $MR(Q)-MC(Q)$  are positive. Consequently, the optimal taxation path cannot lie in these regions, since condition (IV-17a) is violated. Intuitively, in either of these regions, there exists a marginal adjustment to  $Q$  which would simultaneously increase both welfare and collections. That is, the optimal path must lie between these two loci where  $MR(Q)-MC(Q) < 0$  and  $P-MC(Q) > 0$ , so that changing  $Q$  results in a tradeoff between collections and welfare. The feasible region of the optimal taxation path can be limited further by examining equation (IV-17b). Since  $\lambda < 0$ ,  $MB(n)-MC(n)$  and  $MR(n)-MC(n)$  must be of opposite signs for this equation to be satisfied. This eliminates any  $Q, n$  combinations above the  $MR(n)=MC(n)$  locus (since both  $MR(n)-MC(n)$  and  $MB(n)-MC(n)$  are positive in this region) and the region below the  $MB(n)=MC(n)$  locus (since both  $MR(n)-MC(n)$  and  $MB(n)-MC(n)$  are negative in this region). Therefore, condition (IV-17b) requires the optimal path to lie between these two loci (where  $MR(n)-MC(n) > 0$  and  $MB(n)-MC(n) < 0$ ). Consequently, the entire feasible region for the optimal taxation path is represented by  $Q, n$  combinations which lie inside the UTVW region illustrated in Figure IV-4.

As the industry is moved along its optimal path from point  $w$  to point  $T$ , the movements in  $Q$  and  $n$  depend on specific values of parameters and are not determinant for the general case. However, output per firm,  $q$ ,

must fall as one moves along the path toward maximal collections. This could be satisfied by both  $Q$  and  $n$  increasing, both decreasing, or by  $Q$  falling and  $n$  increasing. The latter two possibilities seem intuitively reasonable. Increased collections may require a reduction of both the level of variety and the level of industry sales. Alternatively, the necessary reduction in industry sales may be (optimally) partially offset by promoting a greater variety level. However, the first possibility seems rather surprising. The shaded portion of the feasible optimal taxation region (region UXT) illustrates this surprising possibility that optimal taxation may require both the level of industry sales and the level of variety to be increased as collections are increased. That is, the welfare optimum may have a smaller sales level with less variety than exists when collections are maximized. This is possible because, even though  $Q$  and  $n$  are both less at the welfare maximum than at the tax maximum,  $q$  is larger and, therefore, firms are producing at lower per-unit cost. Consequently, if variety is of little importance to consumers, they may prefer being able to consume fewer goods with less variety at a lower price. This would result if product differentiation were mainly cosmetic and there were substantial economies of scale to be gained by firms operating in the industry.

#### Optimal Taxation Schemes

In this section, we derive those tax instruments required to optimally tax monopolistically competitive industries. In general, at least two taxes are needed to move monopolistically competitive industries to and along their optimal taxation paths. Optimal taxation schemes are derived

for both types of industry behavior (i.e., with and without price competition).

### Nonprice competition

First, consider the case where the industry is characterized by the absence of active price competition. Using equations (IV-6) and (IV-6a), and assuming all three tax instruments are levied, the long-run industry equilibrium is defined by

$$(IV-18a) \quad \pi = P(Q,n)q(1-s) - cq - tq - F - L = 0$$

$$\text{and (IV-18b) } \quad \pi' = (P + P_Q Q)(1-s) - c - t = MR(Q)(1-s) - MC(Q) - t = 0.$$

Equation (IV-18) defines a system of two equations in three unknowns ( $Q$ ,  $n$ , and  $q$ ). This system is made solvable by recalling that (IV-18c)  $Q \equiv nq$ .

The impact that each tax has on output per firm is determined by total differentiation of equation (IV-18), or

$$\begin{bmatrix} P_Q q(1-s) & -P_Q Q(1-s) & P_n q(1-s) \\ (2P_Q + P_{QQ} Q)(1-s) & 0 & (P_n + P_{Qn} Q)(1-s) \\ 1 & -n & -q \end{bmatrix} \begin{bmatrix} dQ \\ dq \\ dn \end{bmatrix} =$$

$$\begin{bmatrix} Pqds + qdt + dL \\ (P + P_Q Q)ds + dt \\ 0 \end{bmatrix} .$$

Then, using the specific demand function defined in equation (IV-2) and applying Cramer's rule yields:

$$(IV-19a) \quad \frac{dq}{dL} = \frac{-1}{(1-s)P_Q Q} > 0$$

$$(IV-19b) \quad \frac{dq}{dt} = \frac{-P_{Qn}}{P_Q(1-s)(P_n + P_{Qn}Q)} > 0$$

$$(IV-19c) \quad \frac{dq}{ds} = \frac{-(PP_{Qn} - P_n P_Q)q}{P_Q(1-s)(P_n + P_{Qn}Q)} > 0.$$

Each of the three tax instruments result in larger output per-firm in the long-run. Intuitively, excise taxes reduce the absolute slope of the net-of-tax demand by reducing the number of firms and pivoting the proportional demand function upward, thereby inducing firms to move down their average cost schedule. License fees increase the gross (of tax) average cost and encourage firms to spread the fixed fee over a larger number of units.

We can now determine the adjustment path of the industry for each form of taxation. A portion of Figure IV-4 is reproduced in Figure IV-5 to aid this analysis. In the absence of taxation, the industry must be located at point N where profits equal zero (i.e.,  $R=0$ ) and each firm is maximizing profits (i.e.,  $MR(Q) = MC(Q)$ ) (from equations (IV-6) and (IV-6a) defining the no-tax long-run industry equilibrium).

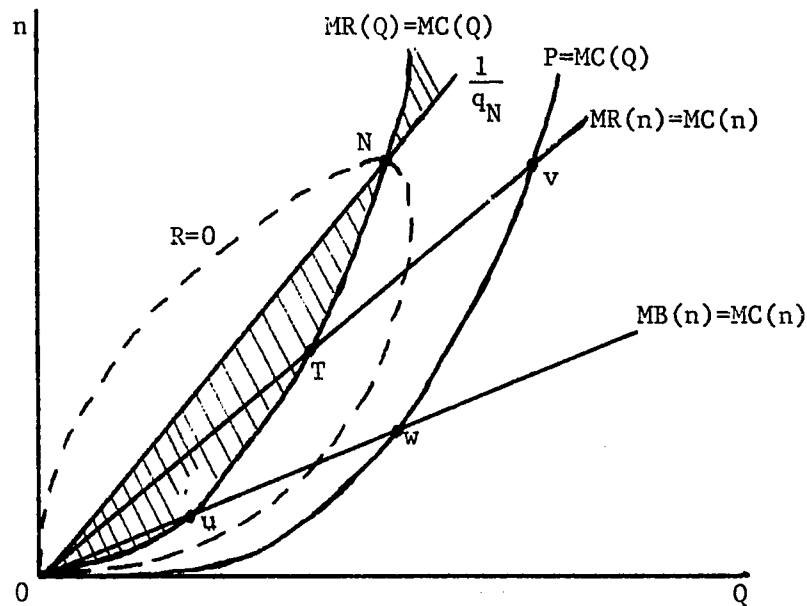


Figure IV-5. Optimal taxation schemes: nonprice competition

Recall that the optimal taxation path lies in the UTVW region. Therefore, we want to find that tax or combination of taxes which move the industry from point N to points inside this region. As taxes are imposed and collections are increased, the industry is moved to collection constraints which lie closer to the point of maximum collections (point T). That is, for any set of taxes, the resulting new equilibrium will lie on a collections constraint inside the  $R=0$  locus. Specifically, it will be that particular collections constraint which keeps net (of tax) industry profits equal to zero or satisfies equation (IV-18a). The particular  $Q, n$  combination (on this new collections constraint) representing the new equilibrium will be that which simultaneously satisfies both equations (IV-18a) and (IV-18b). That is, a new tax-inclusive industry equilibrium is represented

by the intersection of the new (higher level) collection constraint and the  $MR(Q)(1-s)=MC(Q)+t$  locus.

First, consider single unit excise taxes. As excise rates are increased, the industry is moved to collection constraints which lie inside the  $R=0$  locus. From equation (IV-18b), the industry is also moved to a  $Q, n$  combination which satisfies the  $MR(Q)=MC(Q)+t$  locus. This locus must lie above the  $MR(Q)=MC(Q)$  locus, since at any point on the  $MR(Q)=MC(Q)$  locus,  $MR(Q) < MC(Q)+t$ . Consequently,  $MR(Q)$  must be increased by reducing  $Q$  or increasing  $n$  such that  $MR(Q)=MC(Q)+t$ . Therefore, the new equilibrium must lie at  $Q, n$  combinations above the  $MR(Q)=MC(Q)$  locus. Moreover, the new equilibrium point must also lie below the  $1/q_N$  ray. Recall that  $q_N$  represents output per-firm in the absence of taxation. Since excise taxes increase output per-firm (from equation (IV-19b)), the ray originating from the origin and intersecting the new equilibrium point must lie below the initial  $1/q_N$  ray. Therefore, the adjustment path of the industry to unit excise taxes must lie in the shaded region (in Figure IV-5) to points southwest of point N. Similarly, unit excise subsidies move the industry to points in the shaded region to the northwest of point N. Essentially, unit excise tax equilibria are determined by the intersection of the  $MR(Q)=MC(Q)+t$  locus and the  $1/q_c$  ray (where  $q_c$  represents output per-firm under unit excise taxation). First, note that there is no single unit excise rate which can raise maximal potential collections from the industry (i.e., no excise rate can move the industry to point T). Moreover, the adjustment path of the industry to unit excise taxation lies entirely outside the

optimal taxation region. Therefore, single unit excise taxes are not a maximal nor an optimal taxation scheme.

Next, consider single *ad valorem* excise taxes. The adjustment path of the industry to *ad valorem* excise taxes lies in the same regions (but not necessarily on the same path) as under unit excise taxation. Similarly, *ad valorem* tax equilibria are represented by the intersection of the  $MR(Q)(1-s) = MC(Q)$  locus and the  $1/q_s$  ray (where  $q_s$  represents the output level per-firm with *ad valorem* taxes). From equation (IV-19c), the  $1/q_s$  ray must lie below the  $1/q_N$  ray for any positive *ad valorem* rate. At any point on the  $MR(Q) = MC(Q)$  locus,  $MR(Q)(1-s) < MC(Q)$  (given  $s > 0$ ). Therefore, the latter locus must lie above the former locus for any positive *ad valorem* rates. This implies that *ad valorem* excise taxes (subsidies) move the industry to points in the shaded region to the southwest (northeast) of point N as do unit excise taxes (subsidies). Consequently, single *ad valorem* excise taxes cannot maximize collections nor optimally tax the industry.

Finally, consider license fees. Examination of equation (IV-18b) shows that licenses do not affect this marginal condition. License equilibria, therefore, must lie on the  $MR(Q) = MC(Q)$  locus. From equation (IV-19a), positive license fees increase the output level per-firm ( $q_L$ ). License equilibria are represented by the intersection of the  $MR(Q) = MC(Q)$  locus and the  $1/q_L$  ray. Consequently, license fees (subsidies) move the industry along the  $MR(Q) = MC(Q)$  locus to points southwest (northeast) of point N. Contrary to excise taxes, licenses can move the industry to a point on the optimal taxation path. They can move the industry to the

point of maximal collections (point T), but this is the only point on the optimal taxation path that can be reached by license fees.

With the exception of a single license fee which can maximize collections, there is no single tax capable of moving the industry to the optimal taxation path. Moreover, there is no single tax which can move the industry along its optimal path. Use of single taxes in markets characterized by nonprice monopolistic competition are, therefore, generally inappropriate. One can do better with a multiple taxation scheme. Figure IV-5 illustrates that there are three basic multiple taxation schemes which constitute optimal schemes. First, a multiple license fee-excise subsidy scheme can move the industry to and along its optimal path. A license fee can move the industry from point N to points below point T along the  $MR(Q)=MC(Q)$  locus. Then, an excise subsidy can move the industry to points inside the optimal taxation region. Therefore, appropriate rates could move the industry along its optimal path as the collection constraint is increased. Note that a multiple license subsidy-excise scheme cannot be optimal. A license subsidy moves the industry to points above point N along the  $MR(Q)=MC(Q)$  locus. From here, excise taxes (or subsidies) can only move the industry to points which lie totally outside the optimal taxation region. Consequently, a multiple licensing-excise scheme is only optimal if it consists of a license fee and an excise subsidy. Second, consider a multiple excise scheme. Use of both excise taxes can only be optimal if they are of opposite signs. That is, an excise tax (subsidy) moves the industry into the shaded region to points southwest (northeast) of point N, from which an *ad valorem* excise tax (subsidy) can only move the industry



to points further southwest (northeast) in the same region. Clearly, two excise taxes (subsidies) cannot optimally tax the industry. However, a unit excise tax (subsidy) will move the industry to points southwest (northeast) of point N in the shaded region, from which an *ad valorem* subsidy (tax) can move the industry into the optimal taxation region. Finally, the appropriate rates of all three taxes can constitute an optimal taxation scheme. However, the appropriate use of any two taxes is sufficient for optimal taxation.

If the goal is to maximize collections, the simplest method is a single license fee. A multiple excise scheme consisting of a simultaneous tax and subsidy can also move the industry to point T. This requires  $sP + t > 0$  and  $sMR(Q) = t$ , so that  $MR(Q)(1-s) = MC(Q) + t$  coincides with  $MR(Q) = MC(Q)$ . It is interesting that at least two taxes are generally required to optimally tax this industry, while only one tax (a license fee) is needed to maximize collections.

### Price competition

If the industry is characterized by active price competition, then firms do not base output decisions on the true or industry marginal revenue schedule. That is, the  $MR(Q) = MC(Q)$  locus is no longer relevant in this case. Rather, each firm operates on the basis of its perceived marginal revenue schedule ( $MR_p$ ). Equations (IV-6) and (IV-6b) define the long-run price competitive industry equilibrium in the absence of taxation. The industry operates where profits are zero and where perceived marginal revenue equals marginal cost (i.e., where the  $R=0$  locus intersects the  $MR_p = MC(Q)$  locus). The  $MR_p = MC(Q)$  locus is defined by equation (IV-6b) and its slope

is (assuming  $p_q$  is a constant)

$$(IV-20) \quad \left. \frac{dn}{dQ} \right|_{MR_p=MC(Q)} = \frac{-(P_Q n + P_q)}{P_n n - P_q q} > 0.$$

Using equation (IV-20), it can be shown that the slope of the  $MR_p=MC(Q)$  locus exceeds  $1/q$  (i.e., it is concave from above). Moreover, this locus must lie below the  $MR(Q)=MC(Q)$  locus since

$$(IV-21) \quad MR(Q) = P + P_Q Q < MR_p = P + P_q q$$

for any given  $Q, n$  combination. That is, condition (IV-21) is satisfied whenever  $P_Q n < P_q$ , or whenever the absolute slope of the proportional demand exceeds the absolute slope of the perceived demand. In an earlier section, this condition was shown to hold. Also, note that  $P > MR_p$ . Therefore, the  $MR_p=MC(Q)$  locus lies between the  $MR(Q)=MC(Q)$  and  $P=MC(Q)$  loci, as illustrated in Figure IV-6.

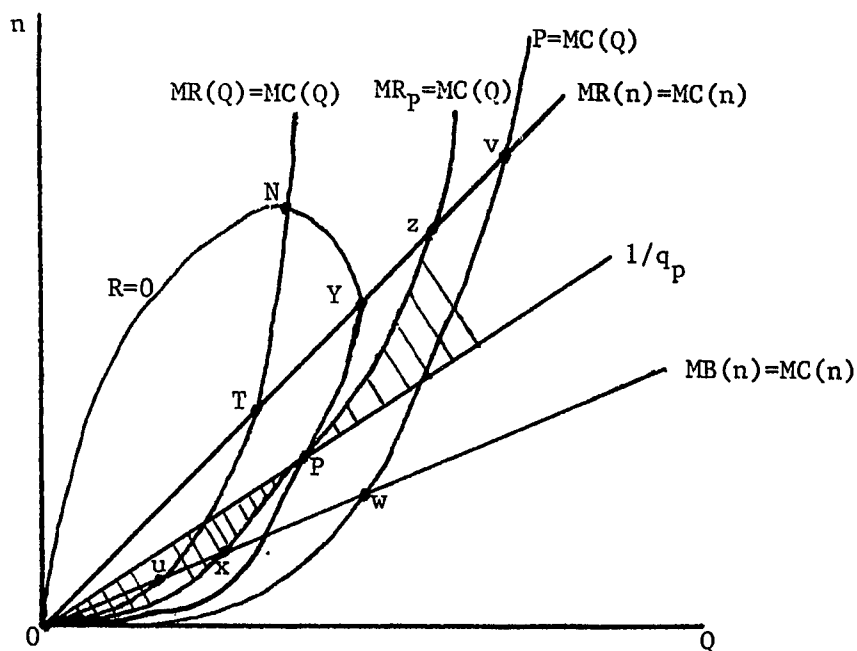


Figure IV-6. Optimal taxation schemes: price competition

From equations (IV-6) and (IV-6b), long-run equilibrium in the absence of taxation is illustrated by point P. Although point P must lie to the right of point N (along the  $R=0$  locus), its actual position depends on the specific values of parameters in industry demand and firms' costs. As drawn, it lies in the optimal taxation region (UTVW). However, it could lie outside this region along the  $R=0$  locus between points Y and N. Let  $q_p$  represent the output per-firm produced in the price competitive equilibrium. Note that  $q_p > q_N$ , which is consistent with our earlier result that price competitors produce a larger output than nonprice competitors.

From equations (IV-6) and (IV-6b), the tax inclusive long-run (price competitive) industry equilibrium is defined by

$$(IV-22a) \quad \pi = P(Q,n)q(1-s) - cq - tq - F - L = 0$$

$$\text{and } (IV-22b) \quad \pi' = (P + P_q)q(1-s) - c - t = MR_p(1-s) - MC(Q) - t = 0.$$

The impact of each separate tax on output per-firm is determined in the same manner as under nonprice competition. Total differentiation of equations (IV-22a), (IV-22b), and (IV-22c)  $Q-nq=0$  (and assuming that  $P_q$  is constant) yields,

$$\begin{bmatrix} P_Q q(1-s) & -P_q q(1-s) & P_n q(1-s) \\ P_Q(1-s) & P_q(1-s) & P_n(1-s) \\ 1 & -n & -q \end{bmatrix} \begin{bmatrix} dQ \\ dq \\ dn \end{bmatrix} = \begin{bmatrix} P_q ds + q dt + dL \\ MR_p ds + dt \\ 0 \end{bmatrix}$$

and, using Cramer's rule:

$$(IV-23a) \quad \frac{dq}{dL} = \frac{-1}{2P_q(1-s)q} > 0, \quad (IV-23b) \quad \frac{dq}{dt} = 0, \text{ and}$$

$$(IV-23c) \quad \frac{dq}{ds} = \frac{q}{2(1-s)} > 0.$$

Both license fees and *ad valorem* excise taxes encourage firms to expand production while unit excise taxes do not alter output per-firm (assuming a linear perceived demand). We can now derive the industry adjustment path for each type of taxation. As rates increase, the industry is moved to collection constraints which lie closer to point T. Tax equilibria are represented by the intersection of the  $MR_p(1-s)=MC(Q)+t$  locus and the  $1/q$  ray (where  $q$  represents the output level chosen by firms when taxes are imposed).

Unit excise taxes do not change output per-firm (by equation (IV-23b)), and, therefore, such tax inclusive equilibria must lie on the  $1/q_p$  ray. For any unit excise tax (subsidy), the  $MR_p=MC(Q)+t$  locus lies above (below) the  $MR_p=MC(Q)$  locus. Therefore, unit excise taxes move the industry from point P toward Point O and unit subsidies move the industry to points northeast of point P along the  $1/q_p$  ray.

From equation (IV-23c), *ad valorem* excise taxes increase output per-firm. Therefore, the industry adjustment path must lie below (above) the  $1/q_p$  ray for *ad valorem* excise taxes (subsidies). For any *ad valorem* excise tax (subsidy), the  $MR_p(1-s)=MC(Q)$  locus lies above (below) the  $MR_p=MC(Q)$  locus. That is, *ad valorem* excise taxes (subsidies) move the industry to points above (below) the  $MR_p=MC(Q)$  locus. Consequently, the industry adjustment path for *ad valorem* excise taxes (subsidies) must lie in the shaded region to the southwest (northeast) of point P in Figure IV-6 --that is, below (above) the  $1/q_p$  ray and above (below) the  $MR_p=MC(Q)$  locus.

Finally, consider the use of license fees. Licenses do not distort the equality between  $MR_p = MC(Q)$  (from equation (IV-22b)). They do, however, increase output per-firm ( $q_L$ ) and pivot the  $1/q_L$  ray below the  $1/q_p$  ray. Consequently, license fees (subsidies) move the industry along the  $MR_p = MC(Q)$  locus to points southwest (northeast) of point P. In summary, unit excises move the industry along the  $1/q_p$  ray, licenses move it along the  $MR_p = MC(Q)$  locus, and *ad valorem* excises move it to points between these two loci.

Whether any single tax can move the industry to a point on the optimal taxation path depends on the position of the initial long-run equilibrium point (P). As drawn, point P lies in the optimal taxation region and could, therefore, constitute the optimal taxation point when  $R=0$ . However, point P could also lie outside the optimal region. Basically, there is nothing fundamental in the price competitive behavior of the industry which ensures that point P lies on the optimal path. First, note that part of the industry adjustment path to license fees does lie in the optimal region regardless of where point P is located. That is, a portion of the  $MR_p = MC(Q)$  locus always intersects the optimal taxation region (UTVW). While license fees can move the industry to a particular point on the optimal path (i.e., for a particular collections level), they cannot move the industry to all optimal taxation points. Excise taxation is very similar. As drawn, either excise tax can move the industry to a particular point on the optimal path, but are generally sub-optimal if used alone. Moreover, if point P lies above point Y, then unit excise taxes are never optimal and *ad valorem* excises may lose the ability to move the industry to even a single point on the optimal path.

In contrast to nonprice competition, use of single taxes in price competitive markets may optimally tax the industry for particular collection levels. However, as in nonprice competitive markets, generally at least two taxes are required to move a price competitive industry along its optimal path. Whether both need to be taxes, or one a tax and the other a subsidy, depends on the particular level of collections and on the location of point P. For example, consider a multiple unit excise-licensing scheme. Assume the objective was to maximize social welfare from the industry (i.e., move the industry to point w). This can be accomplished by a license fee which moves the industry to point x and a unit excise subsidy which (from point x) moves the industry to point w. Alternatively, assume the objective was to maximize collections (or move the industry to point T). This requires a license subsidy to move the industry from point P to point z and an excise tax which moves the industry from point z to point T. Social welfare or tax collections could also be maximized via a multiple excise scheme. For example, an *ad valorem* excise tax could move the industry to a point on the  $MB(n)=MC(n)$  locus, from which a unit subsidy can move the industry to point w. Alternatively, an *ad valorem* subsidy can move the industry to the  $MR(N)=MC(n)$  locus from which collections can be maximized with a unit tax. Therefore, the actual sign of the various tax instruments depends on the specific point on the optimal taxation path to which the industry is moved relative to point P.

Finally, a multiple taxation scheme is also generally required to maximize collections. Contrary to nonprice competition, a single license fee cannot maximize collections, since the  $MR_p=MC(Q)$  locus does not

intersect point T. A single unit excise tax could maximize collections only if point P happened to coincide with point z and *ad valorem* excise taxation only has such potential if point P lies above point Y. Consequently, when the industry is monopolistically (price) competitive, at least two taxes are generally required for both maximal and optimal taxation.

#### Summary: Monopolistic Competition

In general, a multiple taxation scheme is required either to maximize collections or to maximize net welfare given any collections level under conditions of monopolistic competition. However, a single excise tax can accomplish these goals in perfectly competitive markets. Why should these appropriate taxation schemes differ between perfectly and monopolistically competitive markets? In both types of industries, the social welfare from the industry and the collections function are completely defined by the level of industry sales and the number of firms in the industry. Alternatively, since  $Q \equiv nq$ , they are completely defined by the industry sales level and the output level of each firm. Intuitively, there are two targets ( $Q$  and  $q$ ) which must be appropriately adjusted to satisfy the optimal taxation problem. In general, these two targets require two (tax) instruments. That is, one instrument to adjust  $Q$  and one to adjust  $q$  to their appropriate levels. However, in perfectly competitive markets, output per-firm is optimally adjusted in the absence of taxation. That is, each firm operates efficiently at minimum per-unit cost in the absence of taxation. Therefore, a single tax instrument (an excise) is all that is required to adjust the single target,  $Q$  (without disturbing the optimal level of  $q$ ).

In contrast, neither  $Q$  nor  $n$  are generally adjusted appropriately in monopolistically competitive markets without taxation. Two instruments (taxes), therefore, are generally needed to simultaneously adjust the two targets.

Among monopolistically competitive industries, in the absence of price competition, output per-firm is too small in the original long-run equilibrium. Taxes which encourage firms to expand production must be used to move the industry into the optimal taxation region. If firms behave as price competitors, optimal taxation may require either a larger or smaller output per-firm depending on where the original long-run equilibrium is located and on the required collections level. In either case, once output per-firm is appropriately adjusted, the industry sales level (or the number of firms) must also be adjusted.



## CHAPTER V. MONOPOLY

We now proceed to examine optimal taxation in markets characterized by monopoly. It will be assumed that the taxing authority has three tax instruments: unit or *ad valorem* excise taxes and a profit tax. Profit taxes were not considered earlier since free entry into perfectly competitive and monopolistically competitive industries ensures that long-run economic profit is zero. A single-plant monopolist is analyzed; therefore, license fees essentially constitute a profits tax. Moreover, the approach used to examine the optimal taxation path differs from that utilized in previous chapters. The optimal path is described only by the adjustment of industry sales, since the number of firms plays no role given a single-plant monopolist.

In the next section, the optimal taxation path is derived and examined. All points on this path, except the end point corresponding to maximal taxation, require higher output levels than produced by the monopolist in the absence of taxation. The adjustment of the monopolist to various taxes is considered next. Single taxation schemes are shown to be inefficient, except for the singular case of maximal taxation, and the optimal multiple taxation scheme is determined.

## The Optimal Taxation Path

The procedure used to characterize the optimal path in Chapters III and IV is not appropriate for monopoly. First, both the social welfare from the industry and total collections are determined solely by the level of industry sales. We assume the monopolist operates a single plant and

produces a single homogeneous product,  $Q$ . Therefore, the number of firms (or the level of variety) plays no role here. Second, the monopolist may enjoy positive long-run economic profit because of entry barriers. While industry profit is not, therefore, necessarily equal to total tax collections, profits do constitute maximal potential collections for any sales level. That is, industry profit can always be extracted by a 100% profits tax.

Social welfare from the industry and the potential collections function (or industry profits) are defined by the level of industry sales. The optimal taxation path is characterized by the adjustment of industry sales necessary to maximize welfare as required collections are increased. Let  $P(Q)$  (where  $P'(Q) < 0$ ) be the inverse demand function faced by the monopolist where  $Q$  represents sales. The monopolist incurs production cost equal to  $C(Q)$  and marginal costs (MC) are increasing (i.e.,  $MC=C'(Q) > 0$  and  $C''(Q) > 0$ ).<sup>1</sup> Profits ( $\pi$ ) are maximized at that output level where marginal cost equals marginal revenue (MR), or

$$(V-1) \quad \pi = P(Q)Q - C(Q)$$

$$\text{and } (V-2) \quad \pi' = P(Q) + P'(Q)Q - C'(Q) = MR - MC = 0.$$

The long-run equilibrium position of the monopolist in the absence of taxation ( $Q_m$ ) is illustrated in Figure V-1, assuming a linear demand and a quadratic cost structure. The potential tax collections function (industry

---

<sup>1</sup>It is assumed that the private costs of the monopolist are also social costs. In particular, the increasing marginal cost of the monopolist is not the result of the monopolist recognizing some effect that his purchases of inputs have on input prices and rents to suppliers. That is, the monopolist is not an oligopsonist or monopsonist purchaser of inputs for his production process.

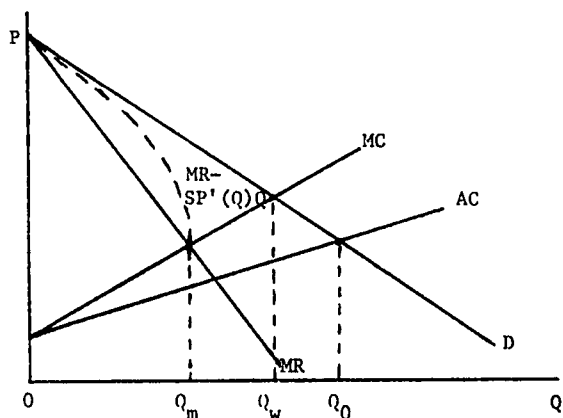


Figure V-1. A single-plant monopolist

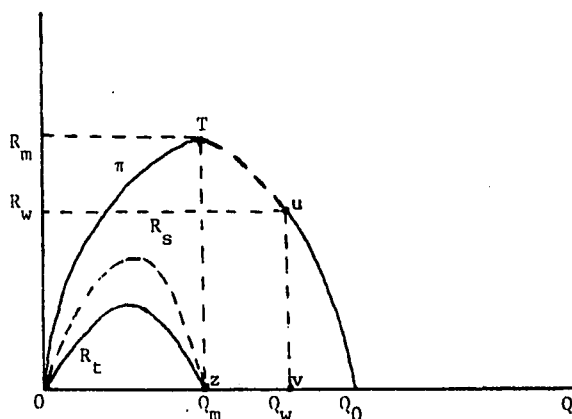


Figure V-2. Single and multiple tax collection functions

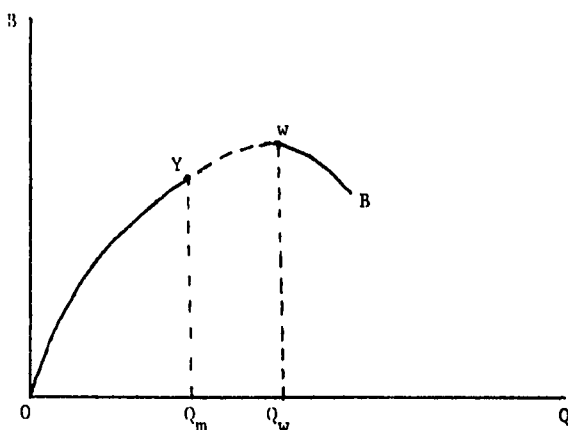


Figure V-3. The social welfare function

profits) is defined by equation (V-1) and illustrated in Figure V-2.

Since the monopolist maximizes his profit in the absence of taxation, potential collections are maximized at  $Q_m$ . No tax collections can be extracted when sales are zero ( $Q=0$ ) or when price equals average cost (AC) at  $Q=Q_0$ . As Figure V-2 illustrates, there are generally two sales levels which yield the same potential collections. The optimal sales level is that which maximizes social welfare from the industry defined by (assuming collections are rebated in a lump-sum fashion)

$$(V-3) \quad B = \int_0^Q P(k)dk - C(Q).$$

Similar to previous analysis, welfare from the industry is equal to gross consumer benefits minus production cost. This function is illustrated in Figure V-3. Its slope equals

$$(V-3a) \quad \frac{dB}{dQ} = P - C'(Q) = P - MC,$$

and it reaches a maximum when  $P=MC$ , illustrated by point  $w$  (where  $Q=Q_w$ ).

The optimal taxation path can be derived by combining Figures V-2 and V-3. First, note that output levels less than  $Q_m$  or greater than  $Q_w$  cannot possibly lie on the optimal path. If output is less than  $Q_m$ , both collections and welfare can be simultaneously increased by expanding sales. Similarly, both collections and welfare are increased by reducing output in excess of  $Q_w$ . The optimal path, therefore, lies between the output level chosen by the monopolist in the absence of taxation ( $Q_m$ ) and the output level where welfare is maximized ( $Q_w$ ). The most that can be collected by taxes is maximal monopoly profit ( $R_m$ ), which leaves the monopolist with

normal returns at point z in Figure V-2. For optimal collection of lesser amounts, output must be increased toward  $Q_w$ . The monopolist continues to earn normal returns (moves from point z to point v in Figure V-2) and collections are reduced toward  $R_w$  along the segment Tu. The expansion of sales increases welfare from point Y to a maximum at point w in Figure V-3. Point u in Figure V-2 and point w in Figure V-3 represent one endpoint of the optimal path. That is, a positive amount of collections is raised as a result of maximizing welfare. For the entire economy, these excess collections can be used to cover the subsidy required to maximize welfare from monopolistically competitive industries.

Formally, points on the optimal taxation path are determined by choosing that sales level, for any given level of collections ( $\bar{R}$ ), which maximizes the following Lagrangean function,

$$(V-4) \quad \mathcal{L} = B + \lambda(\bar{R} - R).$$

Assuming a 100% profits tax is levied, tax collections are equal to monopoly profit. Thus,

$$(V-5) \quad \mathcal{L} = \int_0^Q P(k) dk - c(Q) + \lambda [\bar{R} - P(Q)Q + c(Q)]$$

and it is maximized when

$$(V-5a) \quad \frac{\partial \mathcal{L}}{\partial Q} = P - MC - \lambda(MR - MC) = 0$$

$$\text{and } (V-5b) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - P(Q)Q - C(Q) = 0.$$

Equation (V-5b) simply ensures that the collections requirement is satisfied and equation (V-5a) stipulates that

$$(V-6) \quad \lambda = \frac{P-MC}{MR-MC} .$$

In Chapter II, it was shown that  $\lambda$  must be negative to satisfy the conditions of optimal taxation. This implies that  $P-MC$  and  $MR-MC$  must be of opposite signs which occurs only if output is greater than  $Q_m$  and less than  $Q_w$ . Essentially, taxation schemes are not optimal if they induce the industry to operate on the pareto-inferior side of the tax collections function (i.e.,  $Q < Q_m$ ) or on the pareto-inferior side of the welfare function (i.e.,  $Q > Q_w$ ). Consequently, the optimal path lies between  $Q_w$  and  $Q_m$  which supports our earlier conclusions.

#### Optimal Taxation Schemes

In the absence of taxation, the monopolist operates at the peak of its profit function represented by point T in Figure V-2. With the exception of maximal collections, optimal taxation schemes must induce the monopolist to expand sales. To determine what tax or combination of taxes this requires, we need to examine the adjustment of the monopolist to each form of taxation.

If all three tax instruments are used, monopoly profit is

$$(V-7) \quad \pi = [P(Q)Q(1-s) - C(Q) - tQ] (1-t_\pi)$$

where  $t_\pi$  represents the rate of profit taxation. The monopolist chooses that output where

$$(V-8) \quad \pi' = MR(1-s) - MC - t = 0,$$

such that his net (of tax) profit is maximized. This equation shows how the monopolist reacts to each tax. As is commonly known, profits taxation does not affect the marginal condition of the monopolist. As  $t_{\pi}$  is increased, industry sales remain unaltered at  $Q_m$  and collections are increased from zero at point z to a maximum ( $R_m$ ) at point T in Figure V-2. Since profit taxation does not alter output, it only represents an optimal tax if one desires to maximize collections. Although any potential collection level can be raised with a profits tax, all but the maximum amount could be raised while increasing welfare with an alternative scheme.

Next, consider a single unit excise tax. The per-unit excise rate at any level of sales is determined from equation (V-8) by setting  $s=0$  and solving for  $t$ , or

$$(V-9) \quad t = MR - MC.$$

The unit excise collections function is illustrated in Figure V-2 by  $R_t$ . It originates from  $Q_m$  (where  $MR=MC$ ) and lies everywhere below the monopolist profit function. That is, the per-unit excise rate ( $t = MR - MC$ ) is less than the per-unit profit rate (i.e.,  $P - AC$ ) at every sales level. Figures V-2 and V-3 illustrate the inefficiency of single unit excise taxes. Their revenue potential is less than a profits tax and, since they reduce sales, welfare is less than it is with a profits tax.

The effect of *ad valorem* excise taxes on a monopolists' output is similar to that of unit excise taxes, but the tax collections potential of *ad valorem* excise is larger than unit excises. The effective per-unit *ad valorem* rate (i.e.,  $sP$ ) is determined from equation (V-8) by setting  $t=0$  and solving for  $sP$ , or

$$(V-10) \quad sP = (MR - sP'(Q)Q) - MC.$$

For any positive *ad valorem* rate,  $MR - sP'(Q)Q > MR$  and  $MR - sP'(Q)Q = P$  only if  $s=1$ . That is,  $MR - sP'(Q)Q$  must lie between the marginal revenue and demand schedules as illustrated in Figure V-1. For any sales level, *ad valorem* excise collections ( $R_s$ ) are greater than unit excise collections ( $R_t$ ), but less than gross-of-tax industry profit ( $\pi$ ). That is,  $MR - MC < MR - sP'(Q)Q - MC < P - AC$ , such that  $R_t < R_s < \pi$  which is illustrated in Figure V-2. A higher level of welfare can be obtained with *ad valorem* excise taxes than with an equal-yield unit excise tax. However, either tax results in a lower level of welfare than an equal-yield profits tax.

Use of single excise taxes under conditions of monopoly have an interesting implication for the Laffer hypothesis. Use of either excise can move the industry along an upward-sloping portion of a single excise collections function (i.e., along the "right-side" of a Laffer hill). However, the industry is simultaneously being moved into the "prohibitive range" along the potential multiple-tax collections function. That is, while the taxing authority perceives that it is on the correct side of a Laffer curve, any use of excise taxes alone puts the industry over the Laffer hill into the prohibitive and sub-optimal range along the multiple-tax collections function. Consequently, single excises are never pareto optimal in markets characterized by monopoly. In fact, no single tax can move the industry along its optimal path. Optimal taxation requires a combination of taxes if collections are anything less than maximal.



With the exception of maximal collections, the monopolist must be induced to expand production to move the industry along its optimal path. A profits tax does not alter output and excise taxes reduce it. Therefore, excise subsidies must be used to encourage the monopolist to expand output and operate where  $MC > MR$ . Since excise subsidies cannot possibly satisfy a positive collections requirement, they must be combined with a profits tax. The excise subsidy can expand sales and raise welfare while required collections can be realized by profit taxation.

The subsidy required to expand sales to any level is determined by rewriting equation (V-8) as

$$(V-11) \quad t + sMR = MR - MC.$$

Thus, any excise scheme such that  $t + sMR < 0$  will expand output. For simplicity, assume  $s=0$  and that a single unit excise subsidy is used. Combining a unit excise subsidy equal to  $MC(Q_w) - MR(Q_w)$  with a 100% profits tax yields tax collections of  $R_w$  (i.e., point U in Figure II-2), expands output to  $Q_w$ , yields the monopolist only normal returns (i.e., point v in Figure V-2), and maximizes welfare (point w in Figure V-3). This represents the position of the industry at one endpoint on its optimal path. Reducing the excise subsidy while maintaining a 100% tax on profits moves the industry along the remaining portion of its optimal path. Output falls as the subsidy is reduced and the increase in potential collection is extracted by the profits tax. That is, collections are increased from  $R_w$  (at point u) to a maximum of  $R_m$  (at point T). The 100% profits tax ensures only normal returns for the monopolist (i.e., net monopoly profit moves from point v to point z), and welfare is reduced from a maximum at point w to

point Y. Optimal taxation for all but maximal collections, therefore, requires at least two taxes: an excise subsidy (a unit, an *ad valorem*, or some combination) and a 100% profits tax.

## CHAPTER VI. SUMMARY OF CONCLUSIONS

Most analyses concerning optimal commodity taxation has been restricted to taxation in an economy consisting of only perfectly competitive industries. Since, in such industries, a single excise tax is optimal, the use of other taxes or a combination of taxes has received very little attention. Rather, the major focus has been on whether a uniform or a differential excise rate structure is required. In this paper, we have considered taxation in an economy where both perfectly and imperfectly competitive industries exist together. It was shown that the type of tax or taxes required for optimal commodity taxation is highly dependent on the structure of the market which is taxed. If attention is restricted to perfectly competitive industries, the optimal type of tax is not an issue and the focus has been reduced to the optimal configuration of rates among industries. However, if various types of competitive markets exist, then the optimal choice of tax instruments for each type of industry becomes a major issue which has been the focus of this paper.

For perfectly competitive industries, optimal taxation requires equal proportionate reductions of industry sales and the number of firms so that output per-firm remains constant as collections are increased. Either excise tax or their appropriate combination can accomplish this and move the industry along its optimal path. Licenses by themselves or in combination with excise taxes are not warranted for revenue purposes. They induce a production inefficiency by encouraging firms to expand output. Consequently, their revenue potential is surpassed by excise taxation.

In contrast to perfectly competitive industries, imperfectly competitive industries operate pareto inefficiently in the absence of taxation. Optimal taxation must, therefore, move such industries first onto and then along their optimal paths. For monopolistically competitive industries, the maximization of social welfare requires a net subsidy. The optimal adjustment of the industry from the point of maximal welfare to the point of maximum collections (i.e., the optimal taxation path) depends on the specific characteristics of the market. It may require both the industry sales level and the level of variety (i.e., the number of firms) to fall or the level of variety to increase while the sales level falls as collections are increased. Finally, and surprisingly, optimal taxation may require both the level of variety and the level of industry sales to rise as collections are increased. Moreover, because such industries operate pareto inefficiently in the absence of taxation, it may be possible to raise a substantial level of collections while simultaneously increasing welfare. Whether the market is characterized by price or nonprice competitive behavior, a combination of taxes is generally required to move the industry along its optimal path. Intuitively, both the level of industry sales and the level of variety need to be adjusted appropriately. Therefore, while a single tax may move the industry to a given point on its optimal path, generally at least two taxes are required to simultaneously adjust these two targets. Two taxes are also generally required to maximize collections when the industry is price competitive. However, a single license fee has an unsurpassed revenue potential in nonprice competitive markets.

For markets characterized by a single-plant monopolist, all points on the optimal taxation path (with the exception of the point of maximal

collections) require a larger sales level than produced in the absence of taxation. Collections can be maximized by a 100% profits tax, while all other points on the optimal path require the combination of a 100% profits tax and an excise subsidy. The use of single excise schemes is clearly suboptimal, since they push the industry to the wrong-side of the collections mound.

In general, when the economy is characterized by both perfectly and imperfectly competitive industries, more than one type of tax instrument and a combination of taxes are required. Moreover, while subsidies are not required in perfectly competitive industries, they are needed in a market characterized by a monopoly and may be required in monopolistically competitive industries. Finally, it is interesting to note that, although single taxes can sometimes maximize collections, in most cases a combination of taxes is required for optimal taxation.

## BIBLIOGRAPHY

1. Adams, R. "Tax Rates and Tax Collections: The Basic Analytics of Kahldan-Laffer Curves." *Public Finance Quarterly* 9 (October 1981): 415-430.
2. Atkinson, A.; and J. E. Stiglitz. *Lectures on Public Economics*. New York: McGraw-Hill, 1980.
3. Atkinson, A.; and J. E. Stiglitz. "The Structure of Indirect Taxation and Economic Efficiency." *Journal of Public Economics* 1 (April 1972): 97-119.
4. Atkinson, A.; and J. E. Stiglitz. "The Design of Tax Structures: Direct Versus Indirect Taxation." *Journal of Public Economics* 6 (July-August 1976): 55-75.
5. Baumal, W. J.; and D. F. Bradford. "Optimal Departures from Marginal Cost Pricing." *American Economic Review* 60 (June 1970): 265-283.
6. Bishop, R. "The Effects of Specific and Ad Valorem Excise Taxes." *Quarterly Journal of Economics* 82 (May 1968): 298-218.
7. Bishop, R. "Monopolistic Competition and Welfare Economics." In *Monopolistic Competition Theory*. Edited by Robert Kuenne. New York: John Wiley & Sons.
8. Chamberlin, E. *The Theory of Monopolistic Competition*. Cambridge: Harvard University Press, 1933.
9. Deaton, A. S. "Equity, Efficiency and the Structure of Indirect Taxation." *Journal of Public Economics* 8 (December 1977): 299-312.
10. Diamond, P. A.; and J. A. Mirrlees. "Optimal Taxation and Public Production." *American Economic Review* 61 (March 1971): 8-27.
11. Dixit, A. K. "On the Optimum Structure of Commodity Taxes." *American Economic Review* 60 (June 1970): 295-301.
12. Dixit, A. K. and J. E. Stiglitz. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67 (June 1977): 297-308.
13. Due, J. *The Theory of Incidence of Sales Taxation*. New York: Kings Crown Press, 1942.
14. Henderson, J.; and R. Quant. *Microeconomic Theory: A Mathematical Approach*. New York: McGraw-Hill Book Company, 1980.

15. Higgins, B. "Post-War Tax Policy." *Canadian Journal of Economics and Political Science* 9 (August 1943): 408-428.
16. Lancaster, K. "Socially Optimal Product Differentiation." *American Economic Review* 65 (September 1975): 567-585.
17. Mirrlees, J. A. "Optimal Commodity Taxation in a Two Class Economy." *Journal of Public Economics* 4 (February 1975): 27-33.
18. Pazner, E. A.; and E. Sadka. "Welfare Criteria for Tax Reforms: Efficiency Aspects." *Journal of Public Economics* 16 (August 1981): 113-122.
19. Ramsey, F. P. "A Contribution to the Theory of Taxation." *Economic Journal* 37 (March 1927): 47-61.
20. Sadaka, E. "A Theorem on Uniform Taxation." *Journal of Public Economics* 7 (June 1977): 387-391.
21. Sandmo, A. "A Note on the Structure of Indirect Taxation." *American Economic Review* 64 (September 1974): 701-706.
22. Sandmo, A. "Optimal Taxation; an Introduction to the Literature." *Journal of Public Economics* 6 (July-August 1976): 37-54.

## ACKNOWLEDGMENTS

First and foremost, I wish to thank my major professor, Roy Adams. The many hours he has spent with me, often at night and on the weekends, are far beyond what I could have reasonably expected. Most importantly, he has truly been a friend.

I also wish to express my gratitude to the other members of my committee: Walt Enders, Roy Hickman, Charles Meyer, and Dennis Starleaf. Special thanks also go to Roy Gardner for his helpful comments.

I feel more than thanks for the love and support of my parents and the other members of my family: Heidi, Lynne, Jane, Julie, Laurey, and Mike. You have always been there when I needed you.

Finally, I would like to thank my fiancée, Cindy Anderson, for her love and understanding during this difficult period. Words cannot express the love I feel for you.